

The Large Number of Choices Incompatible with Representing Risk Preference as Utility Curvature

Ned Augenblick* and Seung-Keun Martinez†

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Abstract

EUT has virtually no discriminatory power given two generic lottery decisions (Machina, 1982, 1987). One way to create falsifiable predictions is to assume that the same two decisions are taken across a wealth interval (Rabin, 2000). In this paper, we fully characterize the set of incompatible decisions using this method: if two decisions are associated with non-intersecting intervals of CARA-curvature parameters, there is some wealth-interval at which these decisions cannot be rationalized by *any* increasing utility function. In the simplest example, “risk-seeking” decisions are incompatible with “risk-averse” decisions, but the theory expands this logic to *all* decisions, from risk to prudence to temperance. We also provide examples of decisions that are “fundamentally incompatible” in that a person cannot consistently take the same decision over changing wealth and be an EUM. We then show theoretically and numerically that the required wealth interval for incompatibility is relatively small. We run an experiment to demonstrate the application our theory. We show that 1) decisions are relatively stable across small wealth intervals, 2) curvature intervals between lottery decisions are highly unstable (ie non-rationalizable by EUT), and 3) close to 20% of individuals make “fundamentally incompatible” choices.

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*UC Berkeley Haas. Email: ned@haas.berkeley.edu.

†Defense Resources Management Institute, DSCU / DSCA Email: seungkeun.martinez@nps.edu.

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1 Introduction

This paper aims to better understand the testable predictions of representing risk preferences through the curvature of a utility function. As expected utility theory (EUT) makes few meaningful predictions given two generic decisions, we expand the number of decisions by assuming that a person consistently makes the decisions over a wealth interval (Rabin (2000)).¹ Our main theoretical result is a full characterization of EUT-compatible decisions using this method. The core insight is that lottery decisions that imply different levels of curvature cannot be jointly represented by the same utility function for some wealth interval. We explore how this simple insight places a large amount of structure on allowable decisions and connects preferences over all types of lottery choices. We start with EUT, but discuss how our results generalize to other theories that represent risk through curvature (such as Prospect Theory). We then provide experimental evidence using a set of simple lottery decisions that demonstrate basic violations given the representation of risk through curvature.

The idea of the paper is best understood by first considering a person who prefers a risk-seeking option in one choice while also preferring a risk-averse option in another choice. For an extreme example, suppose that the person consistently prefers a 50% chance of \$100 to a sure \$99 (extreme risk-seeking) while simultaneously preferring a sure \$1 to a 25% chance of \$100 (extreme risk-aversion). Given that one decision is associated with extreme convexity and the other with extreme concavity, the combination appears potentially incompatible with the representation of risk preferences through the curvature of a single function.

Does EUT reject the combination of these decisions? No - it is in fact possible to rationalize these choices with a function that is extremely concave and then becomes extremely convex. More generally, *any* two decisions can be rationalized unless the choices contain a dominance relationship or are related via the independence axiom (which requires that the two choices be related in a very particular way). In other words, risk-as-curvature theories make *virtually no* meaningful testable predictions when studying only two generic isolated choices. Therefore, in order to make predictions and have a falsifiable theory, we must observe additional choices.

There are many different ways to add additional choices. Intuitively, our goal is to capture the notion that the person takes the two decisions *consistently* – that is, continues to take the decision even given a set of different wealth levels (as in Rabin (2000)). For example, suppose that the person does not change their two decisions above when shifting

¹The flexibility and non-falsifiability of EUT has a long history, including Friedman and Savage (1948); Markowitz (1952); Machina (1982); Schoemaker (1982); Machina (1987); Starmer (2000); Zambrano (2005).

their wealth by one dollar (or, equivalently under EUT, when adding one dollar to each of the outcomes). It then turns out that the *four* decisions are not collectively rationalizable with a single increasing utility function. Intuitively, the utility function now needs to be convex and concave in similar regions in a way that is incompatible.² Note that this result (as with our more general results) uses utility at discrete points and therefore does not require any assumptions about continuity or differentiability – only that the utility function is increasing. Proponents of EUT should find this reassuring: although the theory is largely non-falsifiable given two generic decisions, it rejects these intuitively-incompatible decisions when made at different wealth levels.

Given this example, one might hope that establishing incompatibility in the general case simply requires observing two more decisions, each shifted by one dollar. However, this is not the case. The key to the example above is that adding a dollar to each decision allows overlap in the utility components of the different decisions, which allows for a contradiction. Instead, consider another extreme case in which the person prefers a 50.01% chance of \$100 to a sure \$75 (extreme risk-seeking) while simultaneously preferring a sure \$25 to a 49.99% chance of \$100 (extreme risk-aversion). Making these intuitively-incompatible decisions at wealth level w and $w+1$ is allowed under EUT because there is little overlap in the utility terms of the two decisions. It is only when the person consistently makes the decision in $\{w, w+1, \dots, w+25\}$ that the decisions become incompatible with EUT. And, naturally, if we scale the monetary amounts by 100 (so the highest outcome of both choices is \$10,000), then we require the wealth interval to scale to $\{w, w+1, w+2,500\}$. Therefore, giving EUT theoretical bite to reject intuitively-incompatible choices requires that the wealth interval scales with the *length* (the maximum outcome minus the minimum outcome) of the decisions.

This discussion leads to the core question of this paper: which decisions are incompatible with EUT and how large of a wealth interval is required before EUT rejects these decisions? Our main results are that the structure of incompatibility is very simple, the set of incompatible choices is large, and incompatible choices require relatively small wealth intervals before being rejected by EUT. Although some of the statements are simple, the proofs require a large set of seemingly-unconnected mathematics — such as characteristic equations, real and imaginary roots, and sinuodial functions – that we largely attempt to avoid in this introduction to avoid confusion of our main point. As a result, we present the results in more intuitive terms here and push some of the technical caveats into the main text.

To understand the structure of incompatibility, it is again useful to return to the case

²Setting initial wealth to 0, the four choices place restrictions on $u(0), u(1), u(2), u(99), u(100)$ and $u(101)$: $.5 \cdot u(0) + .5 \cdot u(100) \geq u(99)$, $.75 \cdot u(0) + .25 \cdot u(100) \leq u(1)$, $.5 \cdot u(1) + .5 \cdot u(101) \geq u(100)$, and $.75 \cdot u(1) + .5 \cdot u(101) \leq u(102)$. Simple algebraic manipulation implies that $u(1) < u(99)$, which is a violation of dominance.

of simple risk-seeking and risk-averse choices. We first address the case in which a person is *indifferent* between two options that imply risk-seeking, but also *indifferent* between two other options implying risk-aversion. We show that if the person continues to be indifferent across a large enough wealth interval, the person cannot be an expected-utility maximizer (EUM). We then show that the necessary wealth interval is relatively small: a wealth interval that is the length of the largest-length decision will always lead to incompatibility, and, numerically, the necessary wealth interval is much smaller for the large set of decisions we explore.

Then, we move to decisions in which a person can have weak *preference* over choices. Weak preference requires a different (and much longer) proof strategy, but the same basic theoretical result holds: a person cannot consistently prefer a risk-seeking option while simultaneously preferring a risk-averse option in a different decision. While we cannot theoretically pin down a tight bound for the wealth interval, we show numerically that the required wealth interval given preference is very close to that given indifference over a large set of randomly chosen lotteries. That is, incompatibility still happens relatively “quickly” even given the greater flexibility allowed by weak preference rather than indifference.

It is intuitive (and, we believe, theoretically attractive) that EUT would reject a person who is consistently risk-averse and risk-seeking. Our main theorem expands this logic to choices that don’t imply risk-seeking and risk-aversion. To do this, we first need a way to parameterize the curvature associated with different decisions: we define the α -interval of any decision as the set of α s such that the expected utility from the exponential function $-e^{-\alpha \cdot x}/\alpha$ is higher on the chosen decision. For example, the α -interval of preferring \$45 to a 50% chance of \$100 is $[0.004, \infty)$, because $-e^{-\alpha \cdot 45}/\alpha \geq .5 \cdot -e^{-\alpha \cdot 0}/\alpha + .5 \cdot -e^{-\alpha \cdot 100}/\alpha$ for any $\alpha \geq 0.004$. Similarly, the α -interval of preferring a 10% chance of \$100 to \$9 is $(-\infty, 0.002]$. We briefly stress that we are *not* assuming that the person making the decision has an exponential utility function. Our results throughout only assume that the utility function is increasing. Instead, we are using the exponential utility to define the α -interval and provide a particularly-useful measure of the level of curvature associated with different decisions.

Given this parameterization, we can restate the previous result about risk-seeking and risk-aversion as saying that consistently making two decisions where the α -interval of one decision is all above 0 and the interval of the other is all below 0 is incompatible with EUT. Our main result is a generalization of this statement to all other bets: if the intersection of the α -interval of one decision has no intersection with the interval of another decision, then a person who consistently takes those decisions over a big-enough wealth interval cannot be an EUM. That is, just as consistent risk-seeking and risk-aversion are inconsistent with

EUT, consistent “more-curvature-than- $\alpha = .03$ ” and “less-curvature-than- $\alpha = .02$ ” are also incompatible.³

The statement connects choices of all types. Suppose that a person consistently prefers \$40 to 50% (chance) of \$100 for wealth interval $[w, w + 100]$. Then, over the same wealth interval, an EUM cannot consistently prefer 5% of \$100 to \$3.50; or 10% of \$75 to \$6; or 20% of \$200 to \$11; or 25% of \$200 to \$30; or 50% of \$25 and 50% of \$75 to \$48. Similarly, suppose that a person consistently prefers 50% of \$100 to \$35 over the same wealth interval. Then, over that interval, an EUM cannot consistently prefer \$90 to 95% of \$100; or \$110 to 90% of \$150; or \$37 to 80% of \$50; or \$46 to 50% of \$25 and 50% of \$75.

To be clear, we don’t believe that a person having some violations of these very specific bounds is a strong indictment of EUT. Our main point is that one of the fundamental implications of representing risk through the curvature of a single function is that this function is the same across decisions. If decisions are associated with very different curvature, they likely are not coming from the same function. Our suspicion (demonstrated partially by the experiment discussed below) is that the violations are extremely common across many decision types – not just between very small and very large stakes as in Rabin (2000) – to the point that we should seriously question whether curvature is an appropriate representation of risk preferences.

The theorem also applies to more exotic choices. For example, consider the simplest form of a “prudence” choice between a 75% chance of \$20 versus a 75% chance of \$10 and a 25% chance of \$30. Here, both options have the same expected value and variance — the difference is whether the option is right- or left-skewed.⁴ Surprisingly, the α -interval of strictly preferring the first (“imprudent”) option is $\{\emptyset\}$: there is no value of α such that the exponential utility function is higher for the imprudent option. That is, consistent imprudence is *fundamentally incompatible* with EUT. The required wealth interval for incompatibility is relatively small: if the person would pay at least \$1 to get the imprudent option over a wealth interval $[w, w + 50]$, she cannot be not an EUM. And, even if a person prefers the second (“prudent”) option, the level of prudence must be connected to more simple risk decisions. For example, if a person is consistently willing to pay \$1 for “prudent” option, she cannot be risk-averse and consistently prefer \$11 to a 50% chance of \$100.⁵ That is, EUT theory links these two decisions through curvature. However, we suspect (and provide

³We do not say more or less concave/convex because the α -parameter also places restrictions on higher derivatives of the utility function. E.g., how prudent or temperate a person is predicted to be (preferences over the third and fourth moments of a lottery distribution).

⁴“Prudence” is connected to the third-derivative of the utility function, which has been associated with precautionary savings.

⁵Here, the caveat of “risk-averse” is required because a person who is risk seeking would also take the prudent option.

experimental evidence below) that prudence and risk decisions are not linked in that way in real-life decisions.

The same logic holds for “temperance” choices. For example, consider a choice between a 50% chance of \$20 and a 50% chance of \$60 versus a 25% chance of \$0, 50% chance of \$40, and 25% chance of \$80. Here, both options have the same expected value, variance, and skewness — the difference is whether the option is fat-tailed. The theory implies that the EUM’s temperance decision must be connected to her other decisions. For example, if a person consistently prefers the second (“intemperate”) choice, they cannot be consistently risk-averse in a simple risk choice. And, as above, the willingness-to-pay for the temperance choice must be connected to that for simple risk choices.

A corollary expands this logic to a person who is “scale-consistent” in that they take a decision even when the outcomes are scaled by a factor x for some “scale-interval” $[1, x_h]$. In this case, any two decisions with non-intersecting *CRRA*-intervals will be incompatible given a large enough scale interval.

Similar logic holds for other theories that represent risk preferences through a single utility function. For example, suppose a person’s utility is a function of losses or gains relative to their current wealth level. The statements above will not hold because the person’s utility function “resets” given changes in wealth levels. However, fixing the wealth level, the person is using one utility function. In this case, the statements do hold given level-shifts in all of the options. For example, suppose a person prefers \$20 to a 50% chance of \$10 and a 50% chance of \$30 (suggesting risk aversion) and continues to do so when adding $w \in [0, 20]$ to all options (for example, preferring 40 to a 50% chance of \$30 and a 50% chance of \$50). Then, the person cannot prefer a 90% chance of 10 and a 10% chance of 30 to \$12 (suggesting risk seeking) and continue to do so when $w \in [0, 20]$ is added to all options. Our experiment takes this level-shift approach in order to apply more generally.

While we see our contribution as primarily theoretical, the theory implies simple ways to test risk-as-curvature. Rather than having to rely on very specific choices that are related in a particular way (such as independence), risk-as-curvature can be tested using more naturalistic choices. To demonstrate this point, we run a simple preregistered experiment with 505 people on Prolific in which participants are asked to make 10 risky choices given objective lotteries over monetary outcomes.⁶

For example, in the experiment, 86% of people prefer \$20 to a 50% chance of \$10 and a 50% chance of \$30 (risk aversion). However, 69% of people prefer a 90% chance of \$10 and a 10% chance of 30 to \$12 (risk seeking). Given this data, one can immediately conclude

⁶A sample of 356 remains after applying our pre-registered sample restrictions. Our empirical results are not sensitive to this restriction.

that either (1) people’s preferences are not representable by one utility function or (2) their choices must change when some $w \in [0, 20]$ is added to all options. However, when we add \$20 to all of the options, the aggregate levels remain relatively stable at 80% and 70%.⁷ On an individual level, 70% of individuals who make take risk-averse choice in one decision and risk-seeking choice in the other at $w = 0$, do so again at $w = 20$.

Our theoretical results allow us derive precise parametric estimates of the difference in utility curvatures implied by choices. For example, in the experiment we find that, on average, individuals were indifferent between getting \$17.43(\$36.87) with certainty and a lotteries with a 50% chance of \$10(\$30) and a 50% chance of \$30(\$50). With respect to exponential utility, $u(x) = -e^{-\alpha x}/\alpha$, these choices imply an α -interval of $[0.042, 0.054]$.⁸ At the same time, people were, on average, indifferent between getting \$11.03(\$30.29) with certainty and lottery with a 90% chance of \$10(\$30) and a 10% chance of \$30(\$50). These risk-seeking choices corresponds to an α -interval $[-0.067, -0.043]$.⁹ These choices imply significantly different willingness-to-pay when considering a common bet. The α -interval of $[0.042, 0.054]$ implies that someone would be willing to pay \$4.33 to \$4.48 for a 50% chance of getting \$10, but the α -interval of $[0.042, 0.054]$ indicates a WTP of \$5.53 to \$5.82 for the same bet. As such, we find that over wealth interval of 20, that mean behavior is incompatible with the expected-utility representation. These results may remind readers of the violations of EUT from the literature on common consequence and common-ratio effects (Kahneman and Tversky, 1979). This literature finds people systematically over-weigh outcomes with low probability and under-weigh outcomes with high probability, and uses clever pairs of lotteries to induce violations of the independence axiom. In our experiment, we demonstrate similar violations but with simple choices that replicate over wealth.

Our technique also has the advantage of allowing comparisons across decisions over arbitrary lotteries. For example, in our experiment we find people exhibit an average preference for temperance; weakly preferring losing \$0.96 along with a 50% chance of \$20 and a 50% chance of \$40 (50% chance of \$40 and 50% chance of \$60 minus \$1.01) to the fatter-tailed lottery of a 12.5% chance of \$10, a 75% chance of \$30, and 12.5% chance of \$50 (12.5% chance of \$30, a 75% chance of \$50, and 12.5% chance of \$70). This observed WTP for the more temperate (less-tailed) option implies an α -interval of $[0.058, 0.102]$ and WTP of $[\$3.78, \$4.28]$ for a 50% chance of \$10. These choices are again incompatible with expected-utility

⁷Of course, it might be that people switch at some intermediate level (such as $w = 10.50$). But, we would find this to be somewhat bizarre: it suggests that people oscillate between risk-seeking and risk-loving on the same choice given very small amounts of additional money.

⁸To construct this interval, we assume monotonicity (decreasing risk premium) in the certainty equivalent as wealth increases.

⁹In section 5.2 we show these intervals are statistically distinct from each-other with p-value ≤ 0.0001 .

representation when paired with other choices that imply differing levels of curvature — such as the risk-seeking preference for the lottery of a 90% chance of \$10(\$30) and a 10% chance of \$30(\$50) over its expected value.

Perhaps the most striking empirical application is the discovery of choices that are fundamentally incompatible to expected utility representation. Experimentally, we find that 22% of people strictly prefer a negatively skewed lottery, $0.25 \cdot \$10 + 0.75 \cdot \$30$, to a positively skewed lottery, $0.75 \cdot \$20 + 0.25 \cdot \$40$. Of these individuals, 71% also strictly prefer the negatively skewed option when all outcomes are increased by \$20. However this consistently imprudent preference permits no α -interval and is incompatible with EU representation.

Related Literature

As early as Allais (1953) economists have been testing for systematic deviations in observed behavior from the predictions of expected utility theory (Von Neumann and Morgenstern, 1947; Pratt, 1964; Arrow et al., 1965). The most notable critiques of EUT fall into two camps: violations of the independence axiom, often in the form of common-consequence and common-ratio effects (Kahneman and Tversky, 1979; MacCrimmon and Larsson, 1979), and the fact that aversion to small independent gambles leads to either untenable predictions over large-stakes gambles (Rabin, 2000; Barberis et al., 2006) or violations in stochastic dominance when accounting for background risk (Pomatto et al., 2020; Mu et al., 2023).

As a result of these critiques, several alternative theories to EUT have been proposed. Many of these relax the independence axiom so as to accommodate simultaneous risk-seeking and risk-averse choices over lotteries that share a common consequence or a common probability ratio. They often also assume narrow framing since even theories of non-expected utility may require decision makers, who reject a small gamble with positive expected value under background risk, to also reject very favorable large stake gambles (Safra and Segal, 2008).¹⁰ Prominent alternative theories to EUT include prospect theory (Kahneman and Tversky, 1979), rank dependent utility (Quiggin, 1982), cumulative prospect theory (Tversky and Kahneman, 1992), disappointment aversion (Bell, 1982; Loomes and Sugden, 1982; Gul, 1991), reference dependence (Kőszegi and Rabin, 2006), and salience theory (Bordalo et al., 2012).

In relation to the literature that tests the predictions of expected utility theory, our paper combines the intuitions of Rabin’s calibration theorem and papers that show systematic violations of the independence axiom. For example, in Allais’ paradox, he finds that most

¹⁰Narrow framing refers to the phenomenon where independent sources of uncertainty are evaluated separately by the decision maker. For example, if one makes two decisions over two distinct pairs of lotteries, he does not consider the final distribution of outcomes that arises from his two chosen lotteries.

people prefer \$1 million to a gamble that gives an 89% chance of \$1M, a 10% chance of \$5M and a 1% chance of nothing, while also preferring a 10% chance of \$5M to an 11% chance of \$1M. This simultaneously risk-averse and (relatively) risk-tolerant pair of choices famously violates the independence axiom.¹¹ In our work, we are not restricted to the special case of choices that are related by the independence axiom. Rather, if a risk-averse and (relatively more) risk-tolerant choice are consistent across an interval of wealth (à la Rabin (2000)), then that pair of choices cannot be rationalized by EUT. However, in contrast to Rabin (2000), we find in our calculations that the necessary wealth interval to violated EUT is very short, and we are able to provide a characterization of the implied absolute local curvature for all choices between discrete lotteries.

Our work is also conceptually related to the work of Rabin and Weizsäcker (2009), which says that if a person narrowly brackets each choice between lotteries, then either they act as a CARA utility maximizer or will make first-order stochastically dominated choices. This means that the decision maker can act according to the preferences of a standard expected utility maximizer, up to the restriction that they narrowly frame each choice, and still make a combination of choices that is stochastically dominated. In contrast, our work demonstrates that (with or without narrow framing) if an individual consistently makes choices that cannot be rationalized by *some* CARA utility over a sufficiently long interval of wealth, then there does not exist any monotonically increasing utility that can represent their preferences.

Lastly our results can be extended to test the predictions of other models of risk preferences. For example, one can easily adapt the test that we present to models where utility is additively separable across outcomes. That is, models whose distortions relative to EUT take hold over the probability or decision weights on the utility of outcomes, e.g., probability weighting, rank dependence, and salience (Kahneman and Tversky, 1979; Quiggin, 1982; Tversky and Kahneman, 1992; Bordalo et al., 2012). For example, in the standard formulation of cumulative prospect theory, the evaluation of a lottery L with outcomes $\{1, \dots, n\}$ is $CPT(L) = \sum_{n=1}^N w_n \cdot u(n)$, where $w_n = \pi(\sum_{l=1}^n p_l) - \pi(\sum_{m=1}^{n-1} p_m)$ and $\pi(p) = p^\gamma / (p^\gamma + (1-p)^\gamma)^{1/\gamma}$. Here, we can characterize the existence of a utility over outcomes u that rationalizes choice data in an identical manner to our test for EUT, subject to the choice of a probability distortion parameter γ . Alternatively, one could also use our test and the (non)existence of utilities, u , to characterize the set of probability distortions, γ , that rationalize a set of choice data.

¹¹In particular, $\$1M = \$1M (89/100) + [\$1M (100/100)](11/100)$ and $\$1M (89/100) + \$0M (1/100) + \$5M(10/100) = \$1M (89/100) + [\$0M (1/11) + \$5M(10/11)](11/100)$ while $\$1M(11/100) = \$0 (89/100) + [\$1M(100/100)](11/100)$ and $\$5M(10/100) = \$0 (89/100) + [\$0M (1/11) + \$5M(10/11)](11/100)$.

2 Theory Setup

2.1 Environment

We study choices over gain-only finite-outcome lotteries at wealth level w . We assume that raw lottery outcomes are *rational* numbers, so we can rescale all considered outcomes by their least common denominator and work exclusively with integers. That is, we consider $w \in \mathbb{Z}$ and define lottery L as probabilities over integers $\{p_0, 0\}, \{p_1, 1\}, \dots, \{p_m, m\}$, where m is the maximum outcome with a non-zero probability. That is, a lottery taken at wealth w leads to wealth w with probability p_0 , $w + 1$ with p_1 , etc. The *expected value* of lottery L is defined as $\mu[L] = \sum_{i=0}^m p_i \cdot i$.

Utility functions $u(x)$ map outcomes to a real number. Given a utility function u , we define the *expected utility* from lottery L taken at initial wealth level w as $E[u, L; w] = \sum_{i=0}^m p_i \cdot u(w+i)$. Given a decision over two different lotteries L_A and L_B , define $d_i \equiv p_{A,i} - p_{B,i}$ and define the length k of the decision as $\max(i | d_i \neq 0)$.

We say a person is an *expected utility maximizer* (EUM) if her preferences over lotteries can be represented by some utility function: that is, she weakly prefers L_A over L_B at wealth w if and only if $E[u, L_A; w] \geq E[u, L_B; w]$ for some u . Our only assumption on $u(x)$ is that it is increasing: we make no assumptions regarding continuity, differentiability, or concavity. In our analysis, we only consider discrete points $u(0), u(1)$... and make no statements outside of these points.

2.2 Alpha Intervals

It will be useful to parameterize the level of utility-function curvature implied by a given decision. To this end, we use the exponential utility function as a reference utility function $r(x; \alpha) = -e^{-\alpha \cdot x} / \alpha$ (with $r(x; 0) = x$). This utility function becomes more concave as α rises, with $\alpha < 0$ implying convexity; $\alpha = 0$ implying no curvature; and $\alpha > 0$ implying concavity. We stress that we are not assuming that $u(x)$ takes an exponential form, but rather using the exponential form to parameterize the curvature associated with different choices.

We define the α -*intervals* of a decision between two lotteries as the set of α s such that a person with the utility function $r(x; \alpha)$ would take the same decision. That is, the α -interval of a weak preference of L_A over L_B is:

$$\alpha(L_A \succeq L_B) = \{\alpha : E[r(x; \alpha), L_A] \geq E[r(x; \alpha), L_B]\}$$

For example, the α -interval of weakly preferring a 10% chance of \$100 to \$9 is $(-\infty, 0.002]$ because $.9 \cdot -e^{-\alpha \cdot 0}/\alpha + .1 \cdot -e^{-\alpha \cdot 100}/\alpha \geq -e^{-\alpha \cdot 9}/\alpha$ for any $\alpha \leq 0.002$. The α -interval of the opposite weak preference is then $[0.002, \infty)$. Indifference between two non-degenerate choices leads to a discrete set of singleton α values. For example, the α -interval of being indifferent in the choice above is $\{0.002\}$.

2.3 Consistency

Our goal is to derive falsifiable predictions from EUT based on observed choices over lotteries. Almost any pair of decisions given generic lotteries can be rationalized by EUT, except in specific cases involving first-order stochastic dominance (FOSD) or independence. FOSD violations contradict most theories of decision and are weak tests. Independence violations—a hallmark of EUT—have been extensively tested but require very specially constructed pairs where one is a mixture of the other with a common lottery. Thus, EUT makes virtually no falsifiable predictions for two generic non-experimentally-generated choices (Friedman and Savage, 1948; Markowitz, 1952; Schoemaker, 1982; Starmer, 2000; Zambrano, 2005).

To generate testable predictions over a broader space, we consider more decisions by examining choices at varying wealth levels (Rabin, 2000). We say a person *consistently* prefers L_A over L_B over wealth interval $\{w, \dots, w_h\}$ if they prefer L_A to L_B at every (integer) wealth level between w and w_h . Note that, under EUT, this is equivalent to preferring $L_A + \delta$ over $L_B + \delta$ at wealth w for all $\delta \in \{0, \dots, w_h - w\}$, where $+\delta$ adds δ to all outcomes of the lottery.

3 Incompatibility

In this section, we will state our main result and subsequently work through a variety of cases and examples. We start with the case of indifference, as it provides a very tight theoretical bound on the wealth interval needed for incompatibility.

3.1 Indifference

We start with a somewhat esoteric (and measure-zero) restriction on lotteries in the case of indifference: there are no shared complex root pairs of the polynomial $d_k x^k - d_{k-1} x^{k-1} - \dots - d_0$ associated with each decision. This restriction is challenging to interpret economically, but rules out the knife-edge situation where the same *exponentially-damped sinusoid* can keep

the agent indifferent in both choices for long wealth intervals.¹² We state this restriction as $i(L, G) = \emptyset$. Given this restriction, the required wealth interval is very simple:

Proposition 1. *Suppose a person chooses $L_A \sim L_B$ for wealth interval $\{w, \dots, w + l_G\}$ and $G_A \sim G_B$ over wealth interval $\{w, \dots, w + l_L\}$.*

If $\alpha(L_A \sim L_B) \cap \alpha(G_A \sim G_B) = \emptyset$ and $i(L, G) = \emptyset$, then the person is not an EUM.

The formal proof of the Proposition is in the Appendix. The proof is moderately complicated and relies on representing consistent choices through recurrence relations, finding the roots of the associated characteristic equation, and then demonstrating that it is not possible to create a single utility function that can simultaneously be represented as both a linear combination of the roots associated with indifference in the L choice, and indifference in the G choice.

Proposition 1 is *not* stating that the person must have an exponential utility function. Instead, it states that if there is no exponential utility that can rationalize both choices, then there is no increasing utility function that can rationalize them over the wealth interval.¹³ Intuitively, one choice implies that the “average” curvature must be “close” to some amount, while the other choice implies the average must be close to another amount. Both of these statements cannot be true.

The required wealth intervals given indifference are very small

Proposition 1 is tight in the sense that it is always possible to create some utility function (possibly not decreasing) that satisfies both choices if the wealth interval is shrunk slightly. However, it is loose in the sense that it might be impossible to find an *increasing* function, such that the actual interval can be much shorter. For a sense of the actual required wealth interval, we simulate a large number of two random incompatible choices and numerically calculate the wealth interval at which the two choices can no longer be rationalized using EUT (the exact sampling process is described in Appendix Section B).

¹²Consider decision L with lotteries $P_{L_A} = (0.25, 0.25, 0.25, 0.25)$ and $P_{L_B} = (0.20, 0.3998, 0.1002, 0.30)$. The characteristic polynomial has the conjugate root pair $e^{\pm i\pi/50}$. Decision G with $P_{G_B} = (0.20, 0.20, 0.20, 0.20)$ and $P_{G_A} = (0.22, 0.1001, 0.3798, 0.0602, 0.2400)$ is constructed to have the *exact same* complex root pair. Consequently, the utility sequence $u_w = \sin(\frac{\pi}{50} w)$ can rationalize *both* indifferences. This utility function is strictly increasing for the first 25 wealth levels, violating Proposition 1.

¹³That is, while an exponential function with $\alpha = \alpha^*$ rationalizes $L_A \sim L_B$, there are potentially also a continuum of non-exponential functions that will also lead to this behavior that are “close” to this function. For example, if a person is consistently indifferent between \$3 and a 50% chance of \$10, the utility function $r(x; 0.180107)$ will exhibit this behavior, but so will any utility function of the form $r(x; 0.180107) + \epsilon \cdot \sin(2.11591x) \cdot e^{-0.249421 \cdot x}$ for small enough ϵ . Note that this set of utility functions are both increasing and concave.

The distribution of the results are shown in Figure 1 in the dotted lines for three different decision sets. Each distribution represents the wealth interval required (as a percentage of the length) for two random incompatible same-length choices. Note that Proposition 1 requires that all of the results be within 100% of the length. The lightest (leftmost) distributions shows the results when the expected values of the choices in both decisions are constrained to be within 50% of each other. In this case, the required intervals are extremely small - often within 10% of the length of the bet. That is, a person that makes two incompatible decisions with length 100 (such as a 50% chance of \$100 versus \$45) must only consistently do so given a average of \$10 added to their wealth before the decisions cannot be rationalized by EUT. The next two distributions represent the results when the expected values are within 10% and 0.1%, respectively. In this case, while the required wealth intervals are larger, they are still relatively small and all within the length of the lotteries.

3.2 Preference

When a person exhibits weak preference rather than indifference, proving a specific wealth-interval bound becomes much more challenging. However, it is possible to prove that there must be some wealth interval for which incompatible choices imply a violation of EUT:

Theorem 1. *Suppose a person consistently chooses $L_A \succeq L_B$ and $G_A \succeq G_B$ for all w .*

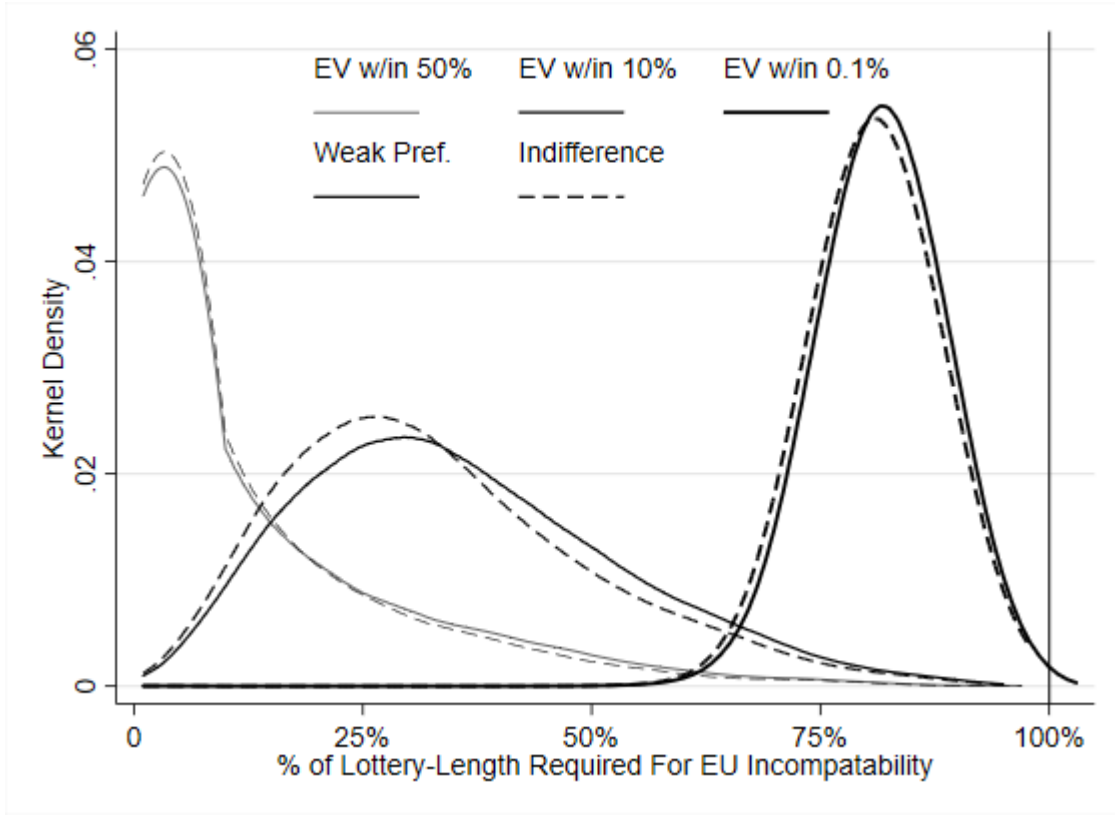
If $\alpha(L_A \succeq L_B) \cap \alpha(G_A \succeq G_B) = \emptyset$, then the person is not an EUM.

Although the result is straightforward, the proof for this theorem is relatively complicated (over 22 pages). Broadly, the challenge is that each decision places restrictions on the “average” curvature over the lottery length, but little restrictions on each individual utility component. By summing over these average restrictions, it is possible to show that, at some point, the implied utility functions must diverge. This strategy is moderately straightforward given “simple” risk-seeking and risk-averse choices. It is harder to generalize to all simple choices (which represent most choices analyzed in the economic literature) and requires somewhat abstract logic to generalize to all choices (in which the α -intervals can take any form).

The required wealth intervals given preference are very small

Unlike Proposition 1, Theorem 1 does not pin down the wealth interval in which decisions become incompatible. Perhaps it is very large? For every choice we have simulated, we have found that the wealth intervals required for incompatibility given preference and indifference

Figure 1: Wealth Intervals Required for Incompatibility



Notes: This figure displays the distribution of wealth intervals required for incompatibility under different constraints given a randomly drawn set of same-length incompatible choices. The x-axis represents the percentage of the wealth interval relative to the length of the bet. The dotted distributions assume indifference given the two choices. The light (left-most) distribution constraints the options to have an expected value within 50% of each other. The medium (center) and dark (right-most) distributions constrain the options to be within 10% and 0.1% of each other. Following Proposition 1, all results are within 100% of the bet length. The solid-line distributions allow for preference and are slightly shifted, with a very small chance of passing 100%.

are *very similar*. For example, in Figure 1, the solid lines represent the distributions of the wealth levels required for incompatibility given preference (versus the dotted lines given indifference). While the wealth level is larger for weak preferences, it is very similar. And, while it is possible to have a wealth interval larger than the length of the lotteries (which is not possible given indifference), it is *extremely rare* in our simulations. For example, even when the two decisions both have choices with expected values within 0.1% of each other, the wealth interval exceeds the lottery length less than 0.001% of the time (and then by a tiny amount).

Intuitively, the results for preference and indifference are similar because the preference is in the “wrong direction” to help create a feasible utility function. For example (and

loosely stated), if one decision implies that the utility function must be convex in many places and other decision implies that the utility function must be concave in many places, it is challenging to create a single utility function that satisfied both conditions — and this challenge is not greatly helped if the function is allowed to be *more* convex and *more* concave.

These results suggest that – minus very strange cases – almost all decisions without overlapping α -intervals are incompatible very quickly, and almost always within the lottery length.

4 Examples

4.1 Simple Decisions

Definition

We define a *simple decision* as those between two lotteries where the CDFs of the two lotteries satisfy the single crossing property. [[CITE - REF?]] Single crossing is satisfied by most lottery-pairs studied in Economics. Without loss of generality, we order A and B such that the CDF of L_A is weakly higher prior to the single crossing point.¹⁴ Given this labeling, we say that L_A is the *risky* option and L_B is the *safe* option.

Simple decisions lead to simple α -intervals. For choices over simple decisions, there is a unique α^* such that $E[r(x; \alpha^*), L_A] = E[r(x; \alpha^*), L_B]$. Then, given our ordering of L_A as the risky option, $\alpha(L_A \succeq L_B) = (-\infty, \alpha^*]$, $\alpha(L_A \preceq L_B) = [\alpha^*, \infty)$, and $\alpha(L_A \sim L_B) = \{\alpha^*\}$. Intuitively, when a person weakly prefers L_A (the risky option), the curvature required is more convex (α is more negative) than α^* , while choosing L_B suggests the curvature is more concave than α^* .

For simple decisions, we define a choice as *risk-averse* when the safe choice L_B has a lower expected value than L_A and the person weakly prefers L_B . In this case, the α -interval of the choice only contains strictly positive numbers. Similarly, we define a choice as *risk-seeking* when the risky choice L_A has a lower expected value than L_B and the person weakly prefers L_A . In this case, the α -interval of the choice only contains strictly negative numbers.

Given these definitions, we state a set of useful corollaries to Theorem 1. The first matches our initial motivation in the introduction:

Corollary 1. *Suppose that a person consistently makes a risk-seeking choice and a risk-averse choice for all w . Then, this person is not a EUM.*

¹⁴Formally, define $F_A(x)$ and $F_B(x)$ as the cumulative distributions of L_A and L_B and $L_{A-B}(x) \equiv F_A(x) - F_B(x)$. Then, our ordering and single crossing occurs if there exists some x^* such that $L_{A-B}(x) \geq 0$ for all $x > x^*$ and $L_{A-B}(x) \leq 0$ for all $x < x^*$.

But, as discussed in the introduction, the same logic applies more generally. For example, for simple decisions with $\alpha^* > 0$, we say a choice with interval $[\alpha^*, \infty)$ implies a risk tolerance less than $R \equiv 1/\alpha^*$ and a choice with interval $(\infty, \alpha^*]$ implies a risk tolerance greater than $R \equiv 1/\alpha^*$. Then:

Corollary 2. *Suppose that a person consistently makes a choice that implies risk tolerance less than R_L and makes a choice that implies risk tolerance greater than $R_H > R_L$ for all w . Then, this person is not a EUM.*

That is, for example, an EUM who consistently prefers \$35 to a 50% chance of \$100 cannot consistently prefer a 10% chance of \$150 to \$7. While the Corollary does not imply the required wealth interval for incompatibility, numerical calculations show it only needs to be \$100 long for these specific choices. The same type of statement can be made for decisions with $\alpha^* < 0$. For example, an EUM cannot consistently prefer a 0.1% chance of \$100 to \$1 and also consistently prefer \$40 to a 10% chance of \$100 (where, again, numerically, a wealth interval of \$100 is all that is needed for incompatibility).

Another way to frame the result for simple decisions is in terms of *indifference-intervals*. To understand the concept, consider a person who is indifferent between \$65 and 75% of \$100 at wealth level w_L ($\alpha = 0.0065$), but whose indifference point monotonically rises to \$67 ($\alpha = 0.0082$) as her wealth increases to w_H . The α -intervals of all of the collective indifference decisions over wealth interval $[w_L, w_H]$ lie in the indifference-interval $[0.0065, 0.0082]$. More formally, consider a decision between $L_A + \Delta$ and L_B . At each wealth level w , there must be some $\Delta(w)$ (possibly negative) such that the person is indifferent between $L_A + \Delta(w)$ and L_B (where $+\Delta$ represents adding the same amount to all outcomes of the lottery). For that wealth level w , define the (singleton) *indifference- α* for the decision between L_A and L_B as $\alpha(L_A + \Delta(w) \sim L_B)$. Finally, define the indifference-interval for decision L_A versus L_B over wealth levels $[w_L, w_H]$ as the smallest interval that contains all indifference- α s over those wealth levels. Then:

Corollary 3. *Suppose that a person's indifference-interval for one decision does not intersect the person's indifference-interval for another decision for all $\{w_L, \dots, w_H\}$. Then, there exists some w_H for which the person is not an EUM.*

Intuitively, if the curvature-level implied by the person's indifference points in one decision is always lower than the curvature-level implied by the indifference points for another decision, then at some point, the necessary differences in curvature are not feasible with a single utility function.

4.2 Exotic Decision-Types

Dominated Choices

If the CDFs of the two lottery options do not cross, then one lottery first-order-stochastically dominates the other. In this case, the choice of the dominant lottery has a corresponding α -interval of $[-\infty, \infty]$, while the dominated choice's α -interval is $\emptyset$: the null set. That is (unsurprisingly), there is no exponential parameter α that can rationalize a dominated choice. We label choices with a null α -interval as *fundamentally incompatible*. Clearly, any dominated choice is incompatible with EUT and requires no additional choices.

Imprudence

Interestingly, dominated choices are not the only fundamentally incompatible choices. For example, consider a “imprudent” choice such as strictly preferring a 75% chance of \$20 versus a 75% chance of \$10 and a 25% chance of \$30. This decision is constructed such that each option has the same expected value and variance. However, although the choice is not dominated, it turns out that the α -interval is $\emptyset$.

Within the proof of Theorem 1, we prove:

Proposition 2. *Suppose that a person consistently makes a fundamentally incompatible choice for all w . Then, that person is not an EUM.*

The wealth interval required for an imprudent choice to become incompatible with EUT depends on the strength of preference. In the example above, if a person is consistently willing to choose the imprudent option even when \$0.20 (\$0.40,\$0.60,\$0.80,\$1.00) is added the prudent choice, the required wealth interval before this person cannot be an EUM is of length \$200 (\$140,\$110,\$95,\$85). Broadly then, EUT does not allow for consistent imprudent choices: if a person seems to have a consistent and real preference for the imprudent choice, they cannot be an EUM.

Temperance

Another previously-researched exotic decision-type is that of *temperance*. A simple version of this decision is between the “temperant” option of a 50% chance of \$25 and a 50% chance of \$75 versus the “intemperant” option of a 25% chance of \$0, a 50% chance of \$50 and a 25% chance of \$100. These options have the same expected value, variance, skewness, but differ in their kurtosis. Given this difference, temperance decisions have been associated

with the fourth derivative of the utility function. However, our results connect temperance choices with other decisions that are traditionally interpreted as being determined by the second derivative. For example, a person cannot consistently be intemperant and also be consistently risk-averse in a simple decision. Similarly, a person cannot be consistently temperant and also be consistently risk-seeking in a simple decision. Finally, the implied curvature across the choices cannot be incompatible: for example, a EUM who is consistently willing to pay at least \$5 for the temperant choice cannot consistently prefer 50% of \$100 to \$35 over a wealth interval of length \$60.

5 Experiment

We next apply our theoretical results to choice data over discrete lotteries. Our theoretical results allow us to characterize the absolute local Bernoulli utility curvature needed to rationalize any decision between discrete lotteries. We know that consistency in utility curvature at different wealth levels for a decision between lotteries L_A and L_B requires that an individual also exhibit the same implied utility curvature for a decision between lotteries G_A and G_B . Additionally, we showed that even in the absence of stochastic dominance there are decisions which are not rationalizable by any monotonic utility if the observed decisions are consistent over multiple wealth levels.

In what follows, we test to what extent observed preferences are consistent between different wealth levels and over different choice types. We measure to what extent an observed preference in one choice type is predictive of the preferences in another, and we test if people display a consistent and robust preference for non-rationalizable choices. Section 5.1 describes our methodology for identifying indifference points between two objective lotteries. These indifference points are then used to create indifference intervals of rationalizing CARA-utility curvature (as discussed in Corollary 3). Section 5.2 presents our analysis of how the implied utility curvature changes between different lottery choices at both the population and individual level.

5.1 Experiment Design

We collected lottery choices from 505 individuals through the survey platform Prolific. Our sample was gender-balanced and only consisted of individuals from the United States who were fluent in English (as measured by Prolific). Participants were given a fixed participation fee—£9 per hour adjusted to median completion time.

Participants were required to make 10 two-part choices. Each choice consisted of a choice

between a Lottery A and a Lottery B. After choosing between lotteries A and B, participants chose how much guaranteed money would have to be *subtracted* from their preferred lottery to make them “equally happy” to play their dis-preferred lottery. This preference elicitation mechanism was designed to identify the point of indifference between two lotteries. Choices were incentivized using a Becker-DeGroot-Marshak payment scheme (Becker et al., 1964).¹⁵ We asked participants how much money would need to be subtracted from their preferred lottery (as opposed to added to their dis-preferred lottery) so the observation of a strict preference between Lottery A and B would not be due to a mistaken belief that the payment scheme could be gamed.¹⁶

For ease of comprehension, we asked participants to imagine they were choosing a Jar from which to draw a ball at random. For example, in Panel A of Figure 2 the participant is asked whether they would rather draw from Jar A, which consists entirely of balls worth \$20, or from Jar B, which has an equal amount of balls worth \$10 and \$30. Panel B of of Figure 2 shows the decision that would follow choosing Jar A over Jar B. The participant is asked how much money would have to subtracted from Jar A for them to be equally happy to draw a ball from Jar B instead. For example, if the participant said \$5 would need to be subtracted, then they would be stating they are indifferent between a (degenerate) lottery that gives \$15 with 100% chance and a lottery that gives \$10 with 50% chance and \$30 with 50% chance. Participants could choose to subtract any amount of money \$X.XX that would not result in negative payoffs.

Each participant made 10 two-part choices similar to the example above. Each of these choices are detailed in Table 1. In each binary choice, participants chose between Jar A and B, before deciding how much guaranteed money would have to be subtracted from their preferred jar to cause them to be indifferent between the two jars. Jars A and B always had the same expected value. We categorize the 10 choices in to five choices types. For each choice type, participants made two decisions: one at “low wealth” and one at “high wealth.” High wealth choices had outcomes that were \$20 more than low wealth choices.

¹⁵For every 2-part decision there was a 50% chance they would be given random draw from their preferred Jar with no money subtracted. There was also a 50% chance of being given a randomly chosen staying price. If the staying price was more than the amount they said would need to be subtracted from their preferred Jar to be equally happy to draw from their non-preferred Jar, their potential bonus was the outcome of a random, computerized draw from their non-preferred Jar. If the staying price is less than the amount they said would need to be subtracted from their preferred Jar to be equally happy to draw from their non-preferred Jar, their potential bonus was equal to the outcome of a draw from their preferred Jar minus the staying price. 1-in-20 participants were randomly selected to have their lottery choices paid out as bonuses.

¹⁶Participants were given in-depth instructions and comprehension testing before making their choices. Participants who got every comprehension correct was paid a \$0.25 bonus. Experiment instructions can be found in Appendix E. They were given choices in increasing level of complexity, but choices were randomized within each level of complexity. The order was risk type 1, risk types 2 and 3 in random order, prudence, and lastly temperance. Low and high wealth choices were shown in random order within in choice type.

Panel B: Indifference Point

You said that you prefer **Jar A** on the previous page.

Panel A: Binary Choice

Consider the Jars below.

Jar A

100% of Balls

\$20

Jar B

50% of Balls

\$10

50% of Balls

\$30

Which Jar would you rather draw one ball at random from?

Jar A
☐

Jar B
☐

Jar A

100% of Balls

\$20

Jar B

50% of Balls

\$10

50% of Balls

\$30

How much **money would need to be subtracted from Jar A** for you to be *equally happy* draw to a ball from Jar B instead of Jar A?

I am equally happy with:

- Jar A – \$

OR

- Jar B

In other words, I am *equally happy* to draw ONE ball from a Jar with:

100% of balls worth \$

OR a jar with:

- 50% of balls worth \$10
- 50% of balls worth \$30

Figure 2: Two-Part Preference Elicitation Example

Notes: This figure gives an example of the 2-part decisions experiment participants were asked to make. In Panel A, the participant had to choose between Jars A and B, representing either getting \$20 with certainty or a lottery that gives \$10 or \$30 with equal probability. In Panel B, shows the decision that would follow choosing Jar A in Panel A. Here, the participant is asked how much guaranteed money would have to be subtracted from Jar A for them to be indifferent between the two jars.

Each of the choice types measured preferences over the second, third, or fourth moment of lottery distributions. That is, each choice type was designed to reveal the implied second, third or fourth derivative of a differentiable Bernoulli utility. The first three choice types all pertained to risk preferences (second moment preferences/second utility derivative). Participants chose how much wealth would need to be lost from either a certain outcome or two-outcome lottery of equal expected value to make them indifferent between Jar A and B.

The prudence choice type elicited preferences over the the skewness of a lottery (third moment preference/third utility derivative). The lotteries obtained from Jars A and B both have the same expected value and variance. However, Jar A is positively skewed and Jar

Table 1: Experiment Lottery Choices

Choice Type	Jar A	Jar B
Risk Type 1	\$20	$\frac{1}{2} \cdot \$10 + \frac{1}{2} \cdot \$30$
	\$40	$\frac{1}{2} \cdot \$30 + \frac{1}{2} \cdot \$50$
Risk Type 2	\$12	$\frac{9}{10} \cdot \$10 + \frac{1}{10} \cdot \$30$
	\$32	$\frac{9}{10} \cdot \$30 + \frac{1}{10} \cdot \$50$
Risk Type 3	\$28	$\frac{1}{10} \cdot \$10 + \frac{9}{10} \cdot \$30$
	\$48	$\frac{1}{10} \cdot \$30 + \frac{9}{10} \cdot \$50$
Prudence	$\frac{3}{4} \cdot \$20 + \frac{1}{4} \cdot \$40$	$\frac{1}{4} \cdot \$10 + \frac{3}{4} \cdot \$30$
	$\frac{3}{4} \cdot \$40 + \frac{1}{4} \cdot \$60$	$\frac{1}{4} \cdot \$30 + \frac{3}{4} \cdot \$50$
Temperance	$\frac{1}{2} \cdot \$20 + \frac{1}{2} \cdot \$40$	$\frac{1}{8} \cdot \$10 + \frac{3}{4} \cdot \$30 + \frac{1}{8} \cdot \$50$
	$\frac{1}{2} \cdot \$40 + \frac{1}{2} \cdot \$60$	$\frac{1}{8} \cdot \$30 + \frac{3}{4} \cdot \$50 + \frac{1}{8} \cdot \$70$

Notes: This table shows all 10 choices experiment participants had to make. As shown in Figures 2, participants would first select if they prefer Jar A or B, then state how much money would have to be subtracted from their preferred jar for them to be indifferent between the two resulting lotteries.

B is negatively skewed. To see this, consider the symmetric lottery: $\frac{1}{2} \cdot \$20 + \frac{1}{2} \cdot \$30$. The low-wealth prudence Jar A then introduces a mean-preserving spread to the higher outcome of $\frac{1}{2} \cdot \$30$ — turning it into $\frac{1}{4} \cdot \$20 + \frac{1}{4} \cdot \$40$. This results in Jar A lottery $\frac{3}{4} \cdot \$20 + \frac{1}{4} \cdot \$40$. Conversely, Jar B introduces a mean preserving spread to the low outcome of $\frac{1}{2} \cdot \$20$ — turning it into $\frac{1}{4} \cdot \$10 + \frac{1}{4} \cdot \$30$. Which results in the Jar B lottery $\frac{1}{4} \cdot \$10 + \frac{3}{4} \cdot \$30$. As such, the choice between Jar A and B reveals whether an individuals prefers to add mean-preserving spread to higher or lower outcomes.

The temperance choice type elicited preferences over the kurtosis (tailedness) of a lottery. The lotteries obtained from Jars A and B are both symmetric (unskewed), and have they the same expected value and variance. However, Jar B temperance lotteries exhibit greater kurtosis. To see this consider the certain outcome of \$30. The low-wealth temperance Jar A spreads $1 \cdot \$30$ into the lottery $\frac{1}{2} \cdot \$20 + \frac{1}{2} \cdot \$40$. Whereas Jar B spreads $\frac{1}{4} \cdot \$30$ into $\frac{1}{8} \cdot \$10 + \frac{1}{8} \cdot \$50$ —resulting in the lottery $\frac{1}{8} \cdot \$10 + \frac{3}{4} \cdot \$30 + \frac{1}{8} \cdot \$50$.

5.1.1 Pre-Analysis Plan and Sample Restriction

We pre-registered this trial with the AEA RCT Registry (AEARCTR-0012180). All of analysis found in Section 5.2 can be found in the uploaded pre-analysis plan. As part of this pre-analysis plan, we stated that we will remove participants from our primary analysis whose choices suggest they may not have understood or paid careful attention to their decisions. Our registered criteria was that we would only retain those who made 2 or fewer stochastically dominated decisions in the second step of their 10 two-part decisions. The results in Section 5.2 accord with this sample restriction. Appendix A replicates our results with two different sample restrictions. In one we retain all participants, and in the other we only retain those who make no dominated choices. Results from retaining only those who make no dominated choices are virtually identical to those found in our primary analysis. Retaining the entire sample only tightens our error bounds.¹⁷

5.2 Experiment Results

Our theoretical work demonstrates that consistency in the implied absolute utility curvature across wealth levels requires that individuals also display consistency in the implied utility curvature across different choice types. In what follows, we examine the between-choice consistency (or lack thereof) in the implied utility curvature between all risk and temperance choices. We exclude prudence choices from this analysis because choosing prudence Jar B cannot be rationalized by any local absolute utility curvature. We separately examine the proportion of individuals choosing prudence Jar B in section 5.2.3, as a consistent preference for Jar B across wealth levels cannot be rationalized by any monotonic Bernoulli utility.

5.2.1 Binary Choice Consistency

We first analyze consistency in the binary choices between Jars A and B. Specifically, we measure the proportion of individuals who stated they preferred the lottery that implies a convex utility function for each choice type. These proportions are depicted in Figure 3. We see that $\geq 80\%$ of individuals would prefer a certain payout of \$20 or \$40 over a 50-50 gamble of $\frac{1}{2} \cdot \$10 + \frac{1}{2} \cdot \$30$ or $\frac{1}{2} \cdot \$30 + \frac{1}{2} \cdot \$50$. Similarly, nearly 90% of people would prefer the certain payments of \$28 or \$48 over gambles of $\frac{1}{10} \cdot \$10 + \frac{9}{10} \cdot \$30$ or $\frac{1}{10} \cdot \$30 + \frac{9}{10} \cdot \$50$. Conversely, only 30% of participants preferred the certain payments of \$12 or \$32 over

¹⁷Table 6 is not reproduced in the Appendix, as we pre-registered a separate sample restriction. We stated we would eliminate outlier values by retaining the inner-fence of observations, as defined by the ± 1.5 -interquartile range.

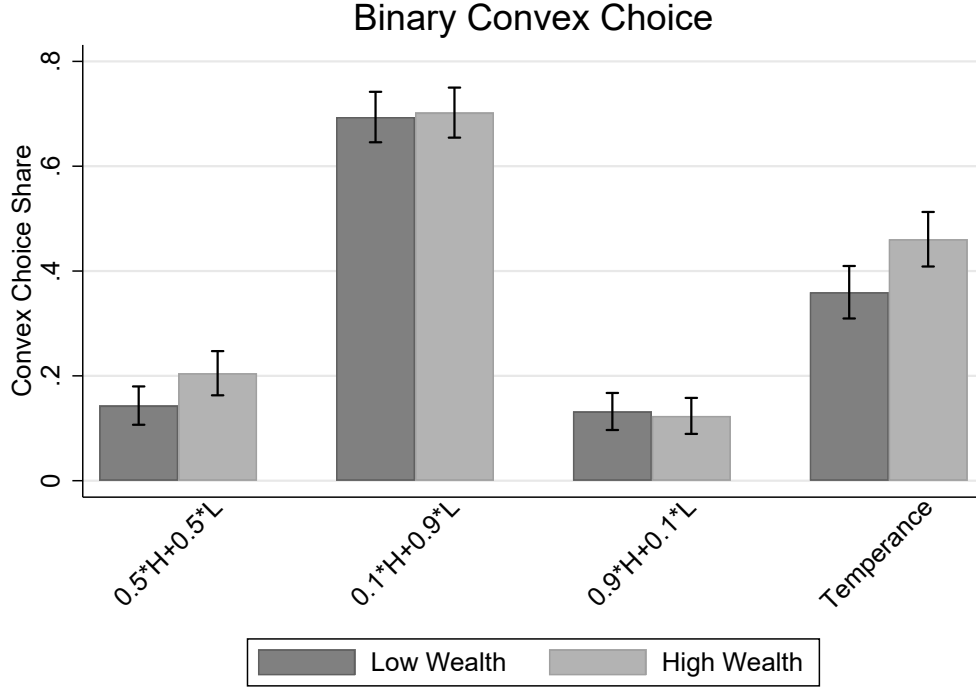


Figure 3: Proportion Choosing Lotteries that Imply Convex Utility

Notes: This figure shows the proportion of individuals choosing Jar B from the lotteries described in Table 1. That is, it shows the proportion choosing a binary lottery over its expected value for risk type choices 1-3, and it shows the proportion if individuals opting for intemperate choices at both low and high wealth.

lotteries of $\frac{9}{10} \cdot \$10 + \frac{1}{10} \cdot \$30$ or $\frac{9}{10} \cdot \$30 + \frac{1}{10} \cdot \$50$. This pattern of risk preferences mimics the widely documented phenomenon of risk aversion when evaluating lotteries with a relatively high chance of a high outcome against its certainty equivalent, and risk tolerance when evaluating lotteries with a relatively low chance of a high outcome against its certainty equivalent. Lastly, we see that, on average, about 60% of individuals prefer 50-50 gambles of $\frac{1}{2} \cdot \$20 + \frac{1}{2} \cdot \$40$ or $\frac{1}{2} \cdot \$30 + \frac{1}{2} \cdot \$50$ over the fatter-tailed gambles of $\frac{1}{8} \cdot \$10 + \frac{3}{4} \cdot \$30 + \frac{1}{8} \cdot \$50$ or $\frac{1}{8} \cdot \$30 + \frac{3}{4} \cdot \$50 + \frac{1}{8} \cdot \$70$.

Tables 2 and 3 allow us to compare how the proportion of individuals making convex utility choices changes across wealth and across choices types. In Table 2 we present the results of regressions of the form

$$y_{i,t} = \gamma_t + \beta_t \cdot H_{i,t} + \epsilon_{i,t}. \quad (1)$$

Where $y_{i,t}$ is an indicator variable for choosing the jar that implies convex utility, and $H_{i,t}$ is an indicator for the choice being made at high wealth. Each of the columns of Table 2 separate the analyzed sample by choice type t . Table 3 presents the δ coefficients of

regressions of the form

$$y_{i,\rho} = \gamma_\rho + \delta_\rho \cdot D_{i,\rho} + \epsilon_{i,\rho}. \quad (2)$$

Where $y_{i,\rho}$ is an indicator for choosing a jar that implies convex utility, and $D_{i,\rho}$ is an indicator for an alternate choice type given a fixed wealth level. Each regression was separated by a pairwise-comparison of choices types given a fixed level of wealth.¹⁸ Each coefficient in Table 3 represents the pairwise difference in the proportion of participants making convex utility choices across choices types for a fixed wealth level.

We see from Table 2 that the proportion of individuals who prefer riskier or less temperate lotteries within a single choice type is relatively stable—the largest change was a 10% increase in the frequency of intemperate choices. However, the proportion of people who seek riskier or less temperate lotteries varies great between choices types—with an additional 50+% of individuals making risk seeking choices when the probability of a high outcome is low.

Table 2: Binary Choice Proportions Across Wealth

	$\frac{1}{2} \cdot H + \frac{1}{2} \cdot L$	$\frac{1}{10} \cdot H + \frac{9}{10} \cdot L$	$\frac{9}{10} \cdot H + \frac{1}{10} \cdot L$	Temperance
	Convex Choice	Convex Choice	Convex Choice	Convex Choice
Wealth Trans.	0.06*** (0.02)	0.01 (0.03)	-0.01 (0.02)	0.10*** (0.03)
Constant	0.14*** (0.02)	0.69*** (0.02)	0.13*** (0.02)	0.36*** (0.03)
Responses	712	712	712	712
Individuals	356	356	356	356

Notes: This table regressing a binary outcome for choosing Jar B from Table 1 (the risk-seeking or intemperate choice) on the wealth level of the choice. *Standard errors, clustered at individual level, shown in parantheses.*

¹⁸That is, $\rho \in \{(.5*H+.5*L, .1*H+.9*L), (.5*H+.5*L, .9*H+.1*L), (.5*H+.5*L, Temp), (.1*H+.9*L, .9*H+.1*L), (.1*H+.9*L, Temp), (.9*H+.1*L, Temp)\} \cdot \{l, h\}$. Where $\rho = (.5*H+.5*L, .1*H+.9*L) \cdot l$ indicates that the regression $y_{i,\rho} = \alpha_\rho + \delta_\rho \cdot D_{i,\rho} + \epsilon_{i,\rho}$ is restricted to choices concerning the lotteries $.5*H+.5*L$ and $1*H+.9*L$ at low wealth.

Table 3: Pairwise Difference in Binary Convex Choice Proportions

	Low Wealth			High Wealth		
	$\frac{1}{2} \cdot H + \frac{1}{2} \cdot L$	$\frac{1}{10} \cdot H + \frac{9}{10} \cdot L$	$\frac{9}{10} \cdot H + \frac{1}{10} \cdot L$	$\frac{1}{2} \cdot H + \frac{1}{2} \cdot L$	$\frac{1}{10} \cdot H + \frac{9}{10} \cdot L$	$\frac{9}{10} \cdot H + \frac{1}{10} \cdot L$
$\frac{1}{2} \cdot H + \frac{1}{2} \cdot L$	0 (-)			0 (-)		
$\frac{1}{10} \cdot H + \frac{9}{10} \cdot L$	0.55*** (0.03)	0 (-)		0.50*** (0.03)	0 (-)	
$\frac{9}{10} \cdot H + \frac{1}{10} \cdot L$	-0.01 (0.03)	-0.55*** (0.03)	0 (-)	-0.08*** (0.03)	-0.58*** (0.03)	0 (-)
Temperance	0.22*** (0.03)	-0.33*** (0.04)	0.23*** (0.03)	0.26*** (0.03)	-0.24*** (0.03)	0.34** (0.03)

Notes: Results of regressing the proportion of individuals who make the convex-preference choice by the choice-type pairs (i.e. choosing Jar B from 1). For example, the coefficient of 0.55*** in the second row of the first column shows that an additiona 55% of participants chose the binary lottery over its expected value when evaluating the lottery of $\frac{1}{10} \cdot \$30 + \frac{9}{10} \cdot \$10$ than when evaluating the lottery of $\frac{1}{2} \cdot \$30 + \frac{1}{2} \cdot \$10$. *Standard errors, clustered at individual level, in parentheses.*

5.2.2 Implied Absolute Utility Curvature

Our binary choice data provides reduced-form evidence of inconsistent curvatures between choices types that would reject a expected-utility representation. In this section, we provide parametric estimates of the local utility curvature implied by participant choices. Our theoretical work outlines how each experimentally elicited indifference point can be associated with set of coefficients of absolute risk aversion (the α -parameter of CARA utility). Using these results, we construct an interval of α -parameters that could rationalize each pair of observed average preferences within a choice type. For example, individuals in our experiment were, on average, indifferent between getting \$17.43 and a the lottery of $\frac{1}{2} \cdot \$10 + \frac{1}{2} \cdot \$30$. At the same time, they were indifferent, on average, between getting \$36.87 and a lottery of $\frac{1}{2} \cdot \$30 + \frac{1}{2} \cdot \$50$. The rationalizing α -interval for this pair of choices is [0.042, 0.054]. Where an EUM with utility $u(x) = -e^{-0.054 \cdot x}$ is indifferent between \$17.43 and a the lottery of $\frac{1}{2} \cdot \$10 + \frac{1}{2} \cdot \$30$, and and EUM with utility $u(x) = -e^{-0.042 \cdot x}$ is indifferent between \$36.87 and a lottery of $\frac{1}{2} \cdot \$30 + \frac{1}{2} \cdot \$50$. Which is to say that participants were, on average, risk-averse for both choices, and, within a wealth interval of \$20, the average coefficient of risk aversion (α -parameter) was within the range of [0.042, 0.054].¹⁹

We construct the rationalizing α -intervals for each pair of high- and low-wealth choices. Figure 4 plots each of these intervals, and Table 4 provides the point estimates for these intervals for all risk and temperance choices.²⁰ In Table 4 we also provide the certainty

¹⁹Here, we are assuming monotonicity in the change of risk-tolerance. As argued before, significant and systematic non-monotonicity over a wealth inter of \$20 would be very strange in its own right.

²⁰While we plot the inferred α -intervals for prudence-related choices, we do not include them in Table 4 or in the analysis of this section. The reason being that the α parameters only exists for prudent choices and

equivalents to a bet that gives \$10 with 50% chance implied by the observed α -parameters for ease of interpretation.

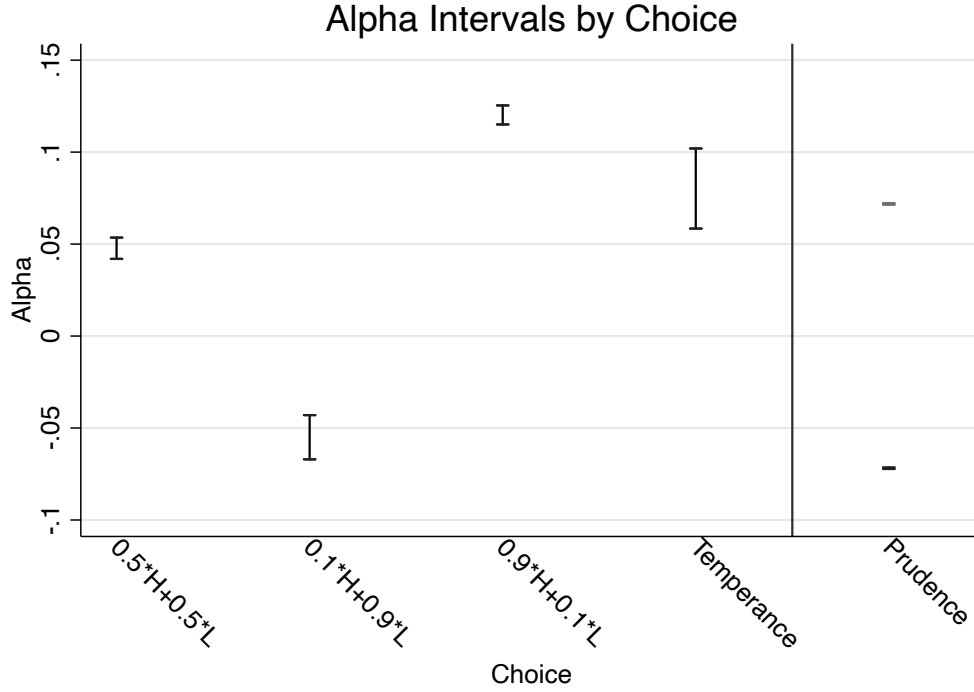


Figure 4: Estimated Mean Alpha by Choice

Notes: This figure plots the α -parameters of the CARA-utility $u(x) = -e^{-\alpha \cdot x} / \alpha$ that would rationalize the mean choices observed in our experiment. For example, experiment participants stated, on average, that they would be indifferent between \$17.43 and a lottery of $\frac{1}{2} \cdot \$10 + \frac{1}{2} \cdot \$30$, and indifferent between \$36.87 and a lottery of $\frac{1}{2} \cdot \$30 + \frac{1}{2} \cdot \$50$. The interval of α -parameters that rationalize these choices is $[0.042, 0.054]$, assuming an individual does not become more risk-averse before becoming less risk-averse over a wealth difference of \$20.

We see from the Figure 4 and Table 4 that the rationalizing α -intervals for mean choices do not overlap—indicating there does not exist a unifying degree of curvature that can rationalize aggregate behavior. Table 5 provides the minimum differences in implied (average) curvature between choice types. For example, we see that when choosing between a $0.5 \cdot H + 0.5 \cdot L$ lottery and its certainty equivalent, individuals, on average, displayed a coefficient of absolute risk averse that was at least 0.085 more risk averse than when choosing between a $0.1 \cdot H + 0.9 \cdot L$ lottery and its certainty equivalent. Table 5 also provides p-values for these estimated differences in curvature coefficients.²¹ We find that implied utility curvature

there exists two rationalizing α -parameters for each prudent indifference-choice. As such, clear comparisons with risk and temperance type choices cannot be made.

²¹The p-values for differences in the edges of the α -intervals were obtained via bootstrapping (Efron and

Table 4: α -Intervals, Certainty Equivalents for $\frac{1}{2} \cdot \$0 + \frac{1}{2} \cdot \$10$

Choice Type	α -Interval	$\frac{1}{2} \cdot \$10$ CE-Interval
$\frac{1}{2} \cdot H + \frac{1}{2} \cdot L$	[0.042,0.054]	[\$4.33, \$4.48]
$\frac{1}{10} \cdot H + \frac{9}{10} \cdot L$	[-0.067, -0.043]	[\$5.53, \$5.82]
$\frac{9}{10} \cdot H + \frac{1}{10} \cdot L$	[0.115,0.125]	[\$3.53, \$3.64]
Temperance	[0.058,0.102]	[\$3.78, \$4.28]

Notes: This table quantifies that α -intervals shown in Figure 4. It also provides the certainty equivalents to a 50% chance of \$10 implied by the α -intervals for a more intuitive comparison.

becomes significantly more convex as the probability of getting the high outcome decreases when considering risk-type bets, $p \cdot H + (1-p) \cdot L$, against their certainty equivalents. In comparing risk-averse choices ($p \cdot H + (1-p) \cdot L$ with $p \geq 0.5$) to temperate choices, participants showed (statistically) nearly-overlapping levels of temperance and risk-aversion when $p = 0.5$, but were more risk-averse than they were temperate when $p = 0.1$.

We next test whether the observed α -parameter from one choice type predicts the α -parameter in another. Recall that EUT requires that the revealed absolute local curvature be consistent between all choice types. Therefore, if someone acts similarly to an EU maximizer, the observed α -parameter in one decision should predict the α -parameter in another. While the results above show a clear divergence in the average α between choice types at a population level, we now ask if, at the individual level, a person's risk aversion over lotteries with a 90% chance of getting the high outcome predicts how risk averse they will be when making choices over 50-50 lotteries. We similarly test if one's temperance in evaluating fat-tailed lotteries predicts his risk aversion.

For this test, we calculated the rationalizing α -parameter for each individually elicited

(Tibshirani, 1994). To perform this bootstrap, we assumed noise occurred at the choice, rather than preference, level—meaning that we re-sampled (with replacement) the average compensating wealth between each Jar A and B 10,000 times and created a distribution of distribution of average α values for each choice based on the re-sampled average compensating wealth amounts. P-values were obtained under the null hypothesis that the edges of two α -intervals were of equal value.

Table 5: Pairwise differences in the Edges of α -Intervals

	$\frac{1}{2} \cdot H + \frac{1}{2} \cdot L$	$\frac{1}{10} \cdot H + \frac{9}{10} \cdot L$	$\frac{9}{10} \cdot H + \frac{1}{10} \cdot L$
$\frac{1}{2} \cdot H + \frac{1}{2} \cdot L$	0 (-)		
$\frac{1}{10} \cdot H + \frac{9}{10} \cdot L$	0.085*** (0.000)	0 (-)	
$\frac{9}{10} \cdot H + \frac{1}{10} \cdot L$	0.062*** (0.000)	0.158*** (0.000)	0 (-)
Temperance	0.012 (0.170)	0.101*** (0.000)	0.013* (0.076)

Notes: This table provides a test of whether the α -intervals in Figure 4 and Table 4 are statistically non-overlapping. That is, whether the distance between the edges of each pair of intervals is statistically larger than 0—indicating there does not exist a single utility function that can rationalize preferences over the two choice types. P-values in parentheses calculated using bootstrap methodology.

point of indifference, and ran the following regressions.²²

$$\alpha_{i,50} = \gamma + \beta \cdot \alpha_{i,90} + \epsilon_i \quad (3)$$

$$\alpha_{i,50} = \gamma + \beta \cdot \alpha_{i,Temp} + \epsilon_i \quad (4)$$

$$\alpha_{i,50} = \gamma + \beta_1 \cdot \alpha_{i,90} + \beta_2 \cdot \alpha_{i,Temp} + \epsilon_i \quad (5)$$

$\alpha_{i,50}$ is the coefficient of absolute risk aversion that rationalizes the compensating wealth that makes person i indifferent between the bet $\frac{1}{2} \cdot H + \frac{1}{2} \cdot L$ and its expected value. $\alpha_{i,90}$ is the coefficient of absolute risk aversion that rationalizes person i 's point of indifference between the bet $\frac{9}{10} \cdot H + \frac{1}{10} \cdot L$ and its expected value. Lastly, $\alpha_{i,Temp}$ rationalizes person i 's point of indifference between Jars A and B in a temperance bet. Observations at both wealth levels were pooled within choice types and standard errors were clustered at the individual level.

We see in Table 6 that while there is a non-zero relationship between the individual-level alpha parameters, these relationships are modest. Further, we see that one's risk aversion in one choice is much more predictive of risk aversion in another, than temperance is predictive of risk aversion. Strict adherence to EUT would require that the pairwise coefficients in columns 1 and 2 be close to one, and that the coefficients in column 3 of Table 6 be nearly colinear.

²²This required that an α -parameter actually exist, so all dominated choices were dropped for this analysis. Additionally, we eliminated outlier α values by limiting the sample to the inner fence (median $\pm 1.5 \cdot$ lower/upper inter-quartile range) of observed values.

Table 6: Individual Alpha Correlations by Choice

	$\frac{1}{2} \cdot H + \frac{1}{2} \cdot L$ (1)	$\frac{1}{2} \cdot H + \frac{1}{2} \cdot L$ (2)	$\frac{1}{2} \cdot H + \frac{1}{2} \cdot L$ (3)
$\frac{9}{10} \cdot H + \frac{1}{10} \cdot L$	0.15*** (0.02)		0.14*** (0.02)
Temperance		0.06*** (0.02)	0.06*** (0.02)
Constant	0.03*** (0.00)	0.04*** (0.00)	0.03*** (0.00)
Responses	710	666	622
R-Squared	0.08	0.02	0.10
Wald Test: Diff. in Coeff. (P-Value)			(0.004)

Notes: We regress α -parameters that rationalizes each individual's certainty equivalents to lotteries of the form $\frac{1}{2} \cdot H + \frac{1}{2} \cdot L$ on the α -parameters that rationalize their certainty equivalents to lotteries of the form $\frac{9}{10} \cdot H + \frac{1}{10} \cdot L$ and their WTP for less-tailed lotteries. Observations over high and low wealth are pooled. Clustered standard errors, at individual level, shown in parenthesis. We keep the inner fence of parameter values.

5.2.3 Unrationalizable Skewness Preferences

Lastly, we examine the proportion of individuals who make imprudent choices. That is, individuals who prefer $\frac{1}{4} \cdot \$10 + \frac{3}{4} \cdot \$30$ over $\frac{3}{4} \cdot \$20 + \frac{1}{4} \cdot \$40$ or prefer $\frac{1}{4} \cdot \$30 + \frac{3}{4} \cdot \$50$ over $\frac{3}{4} \cdot \$40 + \frac{1}{4} \cdot \$60$. There does not exist any CARA utility function that can rationalize these preferences (apart from perfect indifference). This implies that there does not exist any Bernoulli utility that an rationalizes a strict preference for the imprudent option across wealth. Interestingly, we find that at both low and high wealth, 22% of individuals strictly prefer the imprudent option. Lastly, we find significant overlap between those who display strict imprudence at low and high wealth; 71% of individuals who strictly prefer the imprudent option at low wealth also strictly prefer the imprudent option at high wealth.

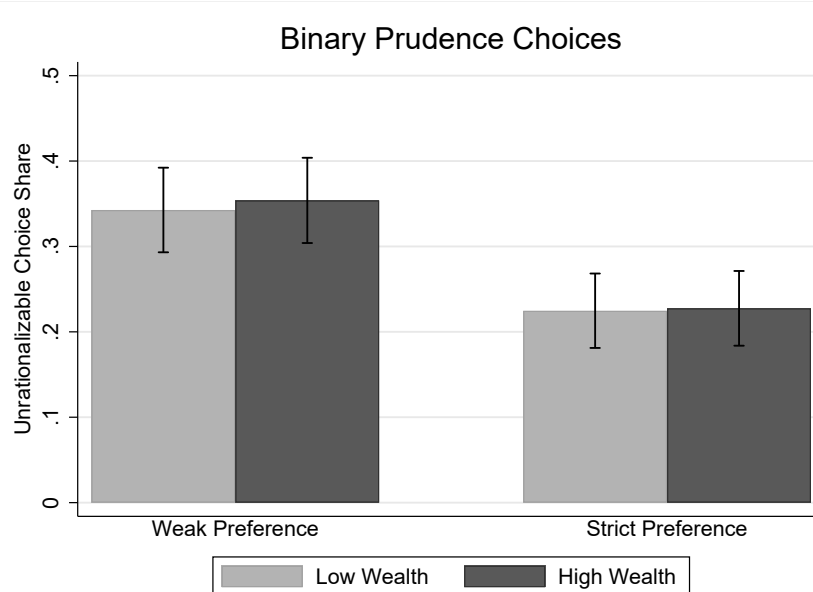


Figure 5: Unrationalizable Imprudent Preferences

Notes: This figure shows the proportion of individuals who make imprudent decisions—preferring lotteries with negative skew. That is, they prefer Prudence Jar B from Table 1. Recall that a consistent strict preference for negatively skewed lotteries is “fundamentally incompatible” with EUT.

6 Conclusion

This paper expands the number of testable implications of the representation of risk through the curvature of a function. The core insight expands that in Rabin (2000): a single utility function cannot lead to decisions that imply different levels of curvature if taken over different wealth levels. This insight places a large and simple structure on the joint set of decisions that are incompatible with EUT. We believe that these results help better understand a core (and testable) implication of EUT. Our broad belief – supported by a set of new experimental results – is that the implied level of curvature across decisions varies widely within an individual. We therefore question the usefulness of representing risk preferences through the curvature of a single function.

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A Experiment Results Appendix

Here we reproduce our experimental results, but with different sample restrictions. In Section A.1 we drop all individuals who many any dominated choice from the dataset. In Section A.2 we keep all observations.

A.1 No Dominated Choices

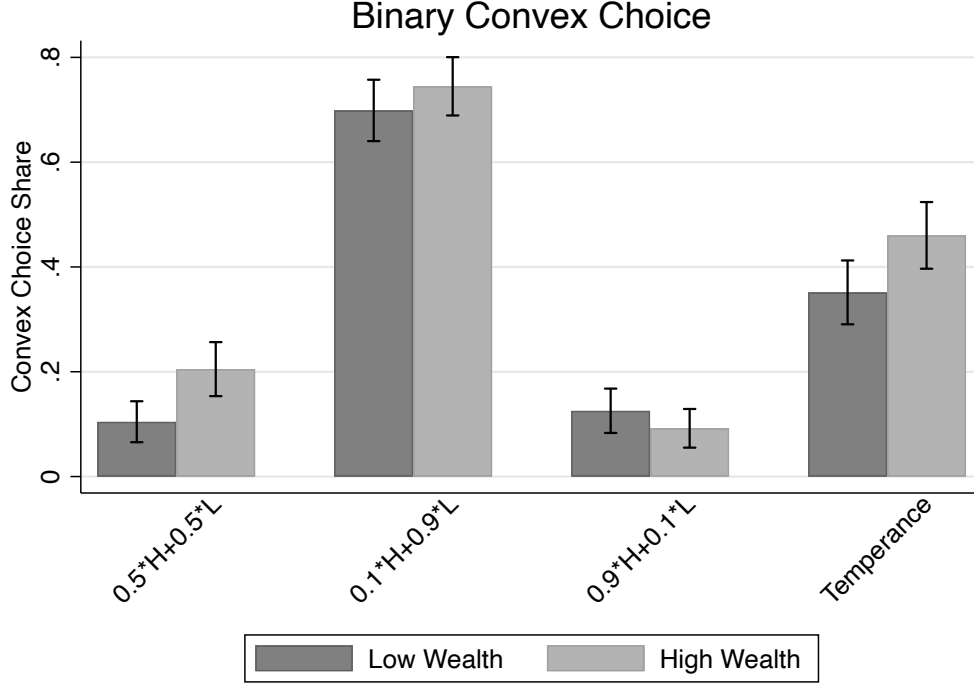


Figure 1: Choices Implying Convex Utility: No Dominated Strict Preferences

Table 1: Alpha Levels by Choice: No Dominated Strict Prefs.

	Convex Choice $\frac{1}{2} \cdot H + \frac{1}{2} \cdot L$	Convex Choice $\frac{1}{10} \cdot H + \frac{9}{10} \cdot L$	Convex Choice $\frac{9}{10} \cdot H + \frac{1}{10} \cdot L$	Convex Choice Temperance
Wealth Trans.	0.10*** (0.03)	0.05 (0.03)	-0.03* (0.02)	0.11*** (0.04)
Constant	0.10*** (0.02)	0.70*** (0.03)	0.13*** (0.02)	0.36*** (0.03)
Responses	478	478	478	478

Notes: Clustered Std Errors at Individual level.

Table 2: Pairwise Difference in Binary Convex Choice Proportions: No Dom. Prefs.

	Low Wealth			High Wealth		
	$\frac{1}{2} \cdot H + \frac{1}{2} \cdot L$	$\frac{1}{10} \cdot H + \frac{9}{10} \cdot L$	$\frac{9}{10} \cdot H + \frac{1}{10} \cdot L$	$5 \cdot H + .5 \cdot L$	$\frac{1}{10} \cdot H + \frac{9}{10} \cdot L$	$\frac{9}{10} \cdot H + \frac{1}{10} \cdot L$
$\frac{1}{2} \cdot H + \frac{1}{2} \cdot L$	0 (-)			0 (-)		
$\frac{1}{10} \cdot H + \frac{9}{10} \cdot L$	0.60*** (0.04)	0 (-)		0.54*** (0.04)	0 (-)	
$\frac{9}{10} \cdot H + \frac{1}{10} \cdot L$	0.03 (0.03)	-0.57*** (0.04)	0 (-)	-0.10*** (0.03)	-0.65*** (0.03)	0 (-)
Temperance	0.25*** (0.03)	-0.34*** (0.04)	0.23*** (0.04)	0.26*** (0.04)	-0.28*** (0.04)	0.37*** (0.04)

Notes: Standard errors, clustered at individual level, in parentheses.

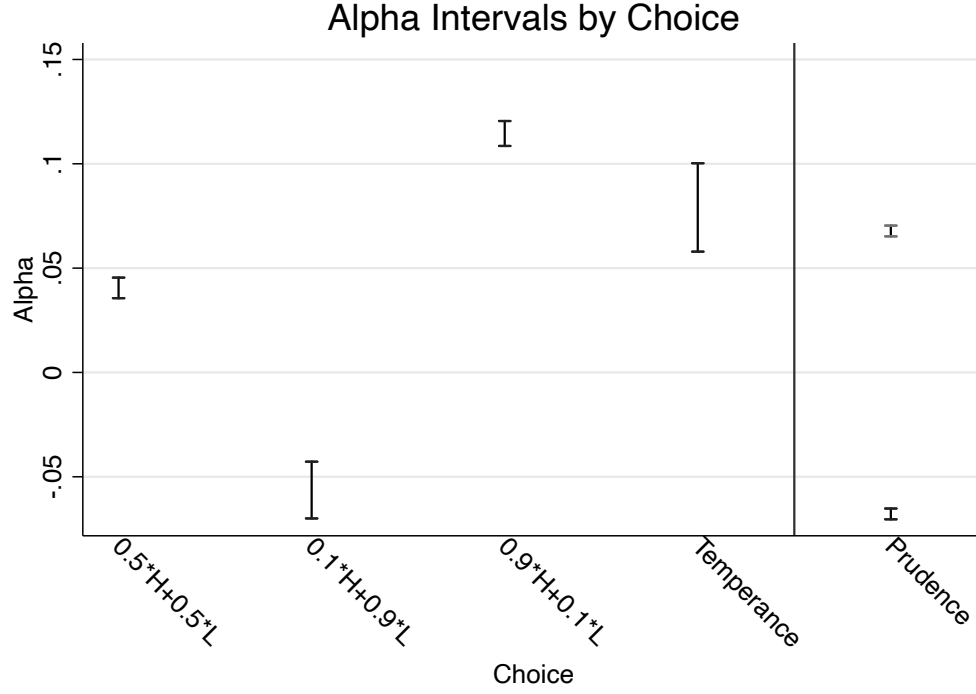


Figure 2: Estimated Mean Alpha by Choice

Table 3: Pairwise differences in the Edges of α -Intervals: No Dominated Prefs.

	$\frac{1}{2} \cdot H + \frac{1}{2} \cdot L$	$\frac{1}{10} \cdot H + \frac{9}{10} \cdot L$	$\frac{9}{10} \cdot H + \frac{1}{10} \cdot L$
$\frac{1}{2} \cdot H + \frac{1}{2} \cdot L$	0 (-)		
$\frac{1}{10} \cdot H + \frac{9}{10} \cdot L$	0.078*** (0.000)	0 (-)	
$\frac{9}{10} \cdot H + \frac{1}{10} \cdot L$	0.063*** (0.000)	0.151** (0.000)	0 (-)
Temperance	0.021 (0.171)	0.109** (0.000)	0.010 (0.191)

Notes: P-values in Parentheses using bootstrap methodology.

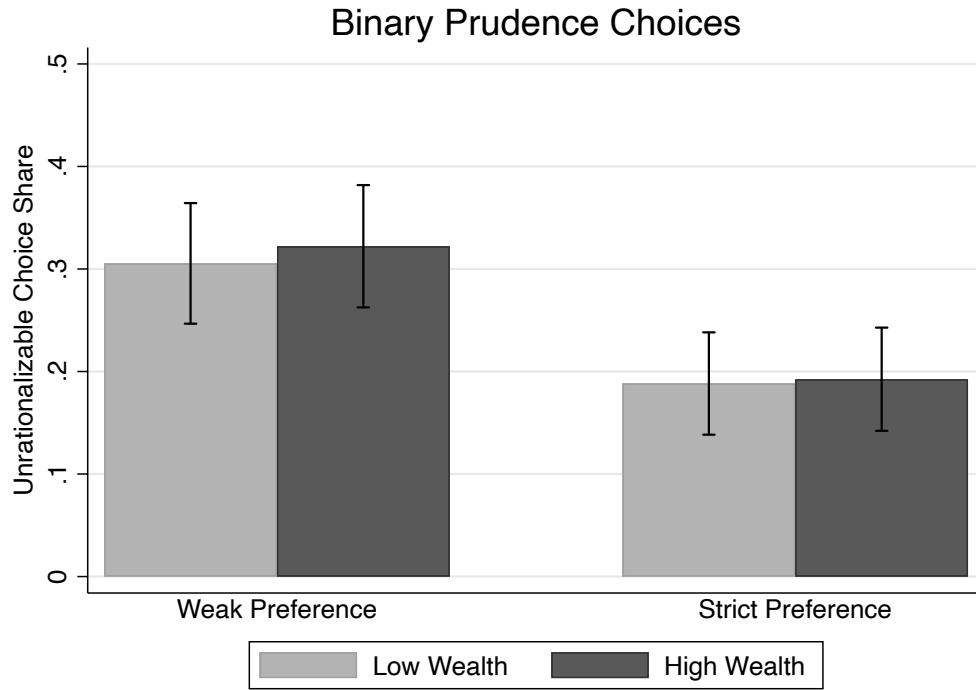


Figure 3: Unrationalizable Imprudent Preferences: No Dominated Strict Prefs.

A.2 All Choices

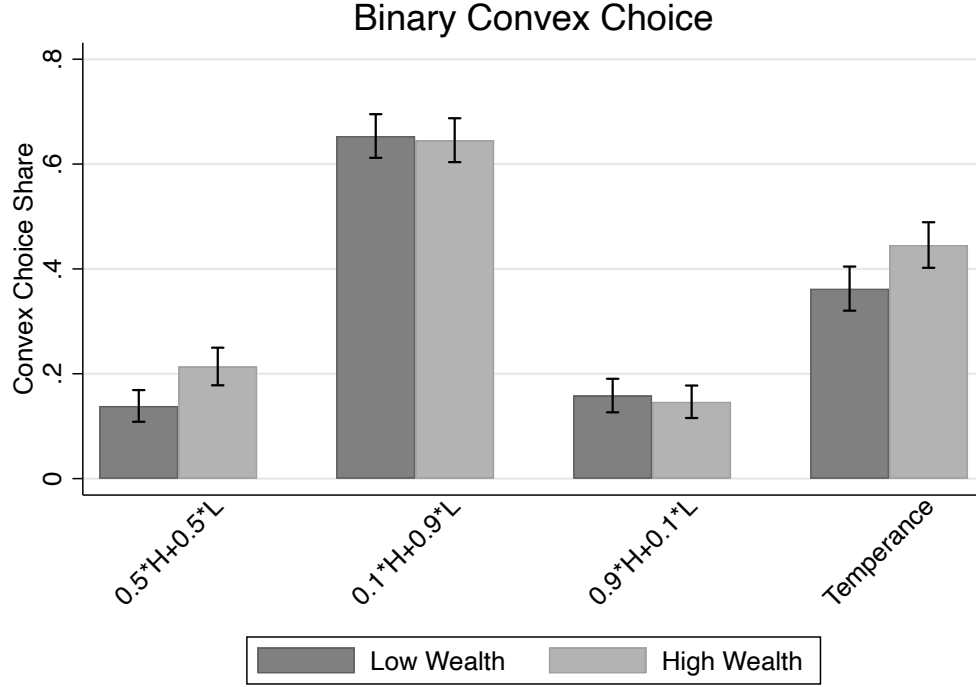


Figure 4: Proportion of Choices Implying Convex Utility: All Observations

Table 4: Alpha Levels by Choice: All Obs.

	Convex Choice $\frac{1}{2} \cdot H + \frac{1}{2} \cdot L$	Convex Choice $\frac{1}{10} \cdot H + \frac{9}{10} \cdot L$	Convex Choice $\frac{9}{10} \cdot H + \frac{1}{10} \cdot L$	Convex Choice Temperance
Wealth Trans.	0.08*** (0.02)	-0.01 (0.02)	-0.01 (0.02)	0.08*** (0.03)
Constant	0.14*** (0.02)	0.65*** (0.02)	0.16*** (0.02)	0.36*** (0.02)
Responses	1010	1010	1010	1010

Notes: Clustered Std Errors at Individual level.

Table 5: Pairwise Difference in Binary Convex Choice Proportions: All Obs.

	Low Wealth			High Wealth		
	$\frac{1}{2} \cdot H + \frac{1}{2} \cdot L$	$\frac{1}{10} \cdot H + \frac{9}{10} \cdot L$	$\frac{9}{10} \cdot H + \frac{1}{10} \cdot L$	$5 \cdot H + .5 \cdot L$	$\frac{1}{10} \cdot H + \frac{9}{10} \cdot L$	$\frac{9}{10} \cdot H + \frac{1}{10} \cdot L$
$\frac{1}{2} \cdot H + \frac{1}{2} \cdot L$	0 (-)			0 (-)		
$\frac{1}{10} \cdot H + \frac{9}{10} \cdot L$	0.51*** (0.03)	0 (-)		0.43*** (0.03)	0 (-)	
$\frac{9}{10} \cdot H + \frac{1}{10} \cdot L$	0.02 (0.02)	-0.50*** (0.03)	0 (-)	-0.07*** (0.02)	-0.50*** (0.03)	0 (-)
Temperance	0.22*** (0.03)	-0.29*** (0.03)	0.20*** (0.03)	0.23*** (0.03)	-0.20*** (0.03)	0.30*** (0.03)

Notes: Standard errors, clustered at individual level, in parentheses.

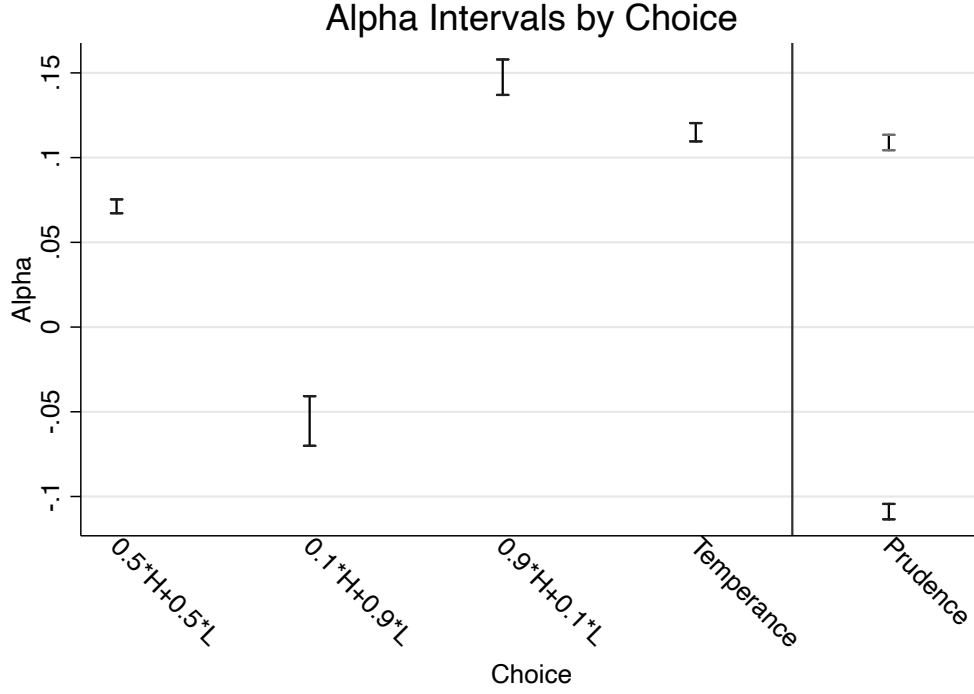


Figure 5: Estimated Mean Alpha by Choice: All Obs.

Table 6: Pairwise differences in the Edges of α -Intervals: All Obs.

	$\frac{1}{2} \cdot H + \frac{1}{2} \cdot L$	$\frac{1}{10} \cdot H + \frac{9}{10} \cdot L$	$\frac{9}{10} \cdot H + \frac{1}{10} \cdot L$
$\frac{1}{2} \cdot H + \frac{1}{2} \cdot L$	0 (-)		
$\frac{1}{10} \cdot H + \frac{9}{10} \cdot L$	0.108*** (0.00)	0 (-)	
$\frac{9}{10} \cdot H + \frac{1}{10} \cdot L$	0.062*** (0.000)	0.178*** (0.000)	0 (-)
Temperance	0.035* (0.052)	0.151*** (0.000)	0.016* (0.074)

Notes: P-values in Parentheses using bootstrap methodology.

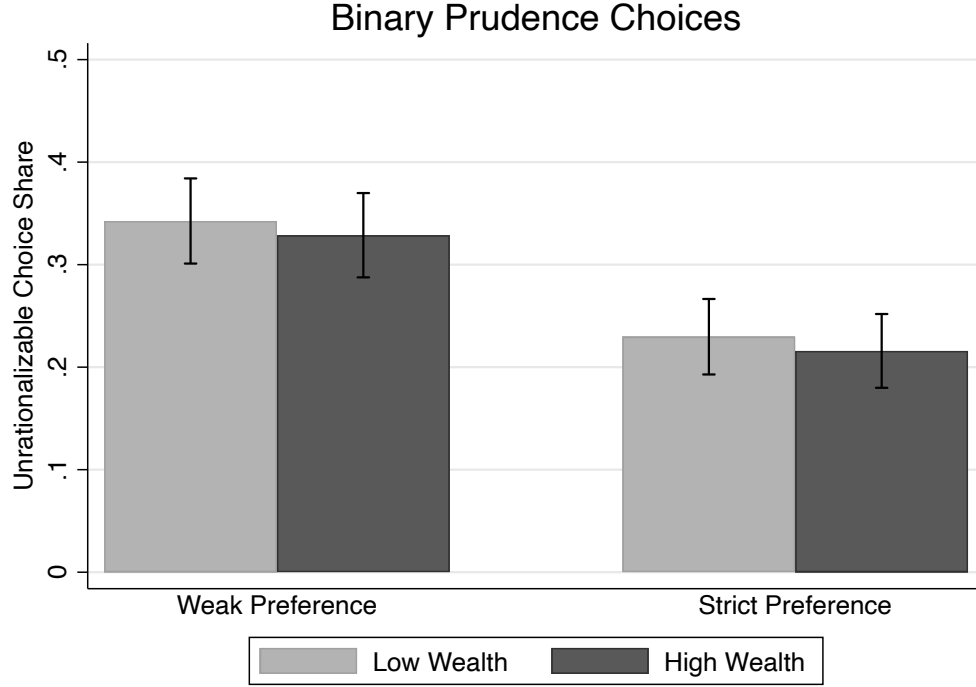


Figure 6: Unrationalizable Imprudent Preferences: All Observations

B Numerical Simulations

In order to determine the wealth interval required before two decisions become incompatible with EUT, we simulate a large set of randomly chosen incompatible lotteries-pairs and determine when incompatibility occurs. For our simulations, the first choice corresponds with risk-seeking and the second with risk-aversion (Lemma ?? about “sister-bets” below implies that the same distributions would result if we were to select from all incompatible simple choices rather than just ones that imply risk-seeking and risk-aversion.)

For our simulations, we start with “very-simple” decisions. A very-simple decision is a decision between a binary lottery and a sure lottery, such as 50% \$0 and 50% \$100 versus \$40. We then mix n very-simple decisions together to create simple decisions. So, for example, mixing the very-simple decision 50% \$0 and 50% \$100 versus \$40 and the very-simple decision 30% \$10 and 80% \$80 versus \$27, leads to the simple decision 25% \$0, 15% \$10, 40% \$80 and 25% \$100 versus 50% \$27 and 50% \$50. Given that the very-simple decisions follow our definition of “simple,” the combination is also “simple.”

To randomly draw a lottery decision, we first draw the number n of very-simple decisions that will be combined from a uniform distribution on $\{0,1,...,10\}$. We then draw the closeness of the expected values of the very-simple decisions from a random uniform draw. We then draw the probabilities and values of the binary lottery for each very-simple decision from a uniform distribution on $[0,100]$. We then calculate the value of the sure lottery from the binary lottery given the previously-drawn closeness parameter (so, for example, if the closeness parameter is .8 and the binary lottery is 50% \$0 and 50% \$100, then we set the sure lottery to be \$40. We then combine all of the n very-simple decision to form a simple decision.

Once we have drawn a simple risk-seeking choice and a simple risk-averse choice, we use a linear programming algorithm (from Mathematica) to determine if there exists a solution (i.e. a utility function) that satisfies the implied constraints for sequentially increasing wealth intervals. We then record the point at which no solution is impossible.

C Proof: Indifference Case

Proof of Proposition 1:

For the Appendix, rather than using the notation $u(n)$ as is standard in Economics, we use u_n given that we only have integer outcomes and we will reframe our analysis in terms of recurrence relationships, where that notation is more common. We refer to each generic value u_n as a “*utility term*”. So, for example, the *expected utility* from lottery L is then

defined as $E[u, L] = \sum_{i=0}^k p_i \cdot u_{w+i}$.

Converting Bets to Recurrence Relations and Roots of Characteristic Equations

In this section, we reframe indifference choices as recurrence relationships and discuss the implications using standard mathematical results. Given that this framing is not common in economics, we proceed at a slow pace. Readers who understand recurrence relationships can likely skip this section.

Suppose that the person is indifferent given the choice between lotteries $L_A = p_0^A 0 + p_1^A 1 + \dots + p_k^A k$ and $L_B = p_0^B 0 + p_1^B 1 + \dots + p_k^B k$, where $p \geq 0$, for wealth levels $w \in \{0, 1, \dots, w_L\}$. This implies a set of *indifference relations*

$$b_0 u_w + b_1 u_{w+1} + \dots + b_k u_{w+k} = 0 \quad (6)$$

for all $w \in \{0, 1, \dots, w_L\}$ where $b_i = p_i^A - p_i^B$ and k is the length of the decision. Rewriting these equalities suggests that they can be summarized in a *recurrence relation*:

$$u_n = c_{k-1} u_{n-1} + c_{k-2} u_{n-2} + \dots + c_0 u_{n-k} \quad (7)$$

for $n \in \{k, k+1, \dots, w_L + k\}$ where $c_i = -b_i/b_k$ (where we may assume WLOG $p_k^B \neq 0$).

A recurrence relation recursively defines a sequence of values u_n given *initial conditions* $u_0, u_1, \dots, u_{k-1}$. That is, given $u_0, u_1, \dots, u_{k-1}$, the relationship in (7) implies u_k , which then implies u_{k+1} , and so on. Recurrence relationships are well-studied outside of Economics and the rest of this section repeats the already-known insights from that literature.

First, we note that equation 7 has a corresponding characteristic equation:

$$x^k = c_{k-1} x^{k-1} + c_{k-2} x^{k-2} + \dots + c_0. \quad (8)$$

The Fundamental Theorem of Algebra states that this equation will have k roots. Label these roots as $\lambda_1, \dots, \lambda_k \in \mathbb{C}$. We may then use these roots to represent the utility that solves the equality relationship in (6) for all utility terms u_n with $n \leq w_L + k$:

$$u_n = d_1 \cdot (\lambda_1)^n + d_2 \cdot (\lambda_2)^n + \dots + d_k \cdot (\lambda_k)^n \quad (9)$$

where $\{d_1, \dots, d_k\}$ are given by initial conditions. Note that $x = 1$ is always a root because $\sum_{i=0}^{k-1} c_i = 1$.

For simplicity in what follows, assume the characteristic polynomial has k distinct roots. With repeated roots, the standard $n^m \lambda^n$ terms appear, and the statements adapt by using

a generalized Vandermonde.

It will be useful to use a matrix formulation to describe the relationship and to define $\{d_1, \dots, d_k\}$ from a set of initial conditions. Specifically, (7) implies a useful matrix relationship in which a *companion matrix* C “shifts” a vector of utility values:

$$\begin{bmatrix} u_{n-k+1} \\ u_{n-k} \\ \dots \\ u_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \\ c_0 & c_1 & c_2 & \dots & c_{k-1} \end{bmatrix} \begin{bmatrix} u_{n-k} \\ u_{n-k+1} \\ \dots \\ u_{n-1} \end{bmatrix} \quad (10)$$

for $n \in \{k, k+1, \dots, w_L + k\}$. Or, in more compact notation:

$$\vec{u}_{n-k+1,n} = C \cdot \vec{u}_{n-k,n-1} \quad (11)$$

This will be a useful formulation because the transformation can be repeated:

$$\begin{aligned} \vec{u}_{n-k+2,n+1} &= C \cdot \vec{u}_{n-k+1,n} \\ &= C \cdot (C \cdot \vec{u}_{n-k,n-1}) \\ &= C^2 \cdot \vec{u}_{n-k,n-1} \end{aligned}$$

Or, more generally, starting with *initial conditions* $\vec{u}_{0,k-1}$:

$$\vec{u}_{n-k+1,n} = C^{n-k+1} \cdot \vec{u}_{0,k-1} \quad (12)$$

Given the repeated transformation, it will be useful to decompose the matrix C . Specifically:

$$C = V \cdot \Lambda \cdot V^{-1} \quad (13)$$

where:

$$V \equiv \begin{bmatrix} 1 & 1 & \dots & 1 \\ \lambda_1 & \lambda_2 & \dots & \lambda_k \\ \dots & \dots & \dots & \dots \\ \lambda_1^{k-1} & \lambda_2^{k-1} & \dots & \lambda_k^{k-1} \end{bmatrix} \text{ and } \Lambda \equiv \begin{bmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \lambda_k \end{bmatrix}, \quad (14)$$

and λ_i s is the i th (of k) eigenvalues of C . The λ s can be found as the roots of the characteristic equation 8 described above.

But, then (12) can be rewritten:

$$\begin{aligned}\vec{u}_{n-k+1,n} &= (V \cdot \Lambda \cdot V^{-1})^{n-k+1} \cdot \vec{u}_{0,k-1} \\ &= V \cdot \Lambda^{n-k+1} \cdot V^{-1} \cdot \vec{u}_{0,k-1}\end{aligned}\tag{15}$$

$$\equiv V \cdot \Lambda^{n-k+1} \cdot \vec{d}\tag{16}$$

where we introduce the definition of the *constants vector* $\vec{d}$ in the final line. The simple form of $V \cdot \Lambda^i$ leads to a simple solution for all utility terms given $n \leq w_L + k$:

$$\vec{u}_{0,n} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ \lambda_1 & \lambda_2 & \dots & \lambda_k \\ \dots & \dots & \dots & \dots \\ \lambda_1^n & \lambda_2^n & \dots & \lambda_k^n \end{bmatrix} \cdot \vec{d}\tag{17}$$

$$\equiv V_{0,n} \cdot \vec{d}\tag{18}$$

where again the last line introduces the definition for the $(n+1)$ -by- k matrix $V_{0,n}$ (note that the previously-defined square-matrix V is equal to $V_{0,k-1}$). In slightly more readable terms, we now state a slightly variation on a well-known Proposition about recurrence relations:

Statement 1. *Given the equality relationships in (6), all utility terms u_n with $n \leq w_L + k$ can be represented in the form:*

$$u_n = d_1 \cdot (\lambda_1)^n + d_2 \cdot (\lambda_2)^n + \dots d_k \cdot (\lambda_k)^n\tag{19}$$

for constants $d_1, \dots, d_k$.

That is, all utility terms following the *initial conditions* $\vec{u}_{0,k-1}$ can be calculated using a linear combination of the eigenvalues of the characteristic equation (8), with the weights determined by the linear projection of the initial conditions onto the matrix V . λ_i may be real or complex, and that complex roots come in conjugate pairs. If $\lambda = \alpha \pm i\beta$ is such a pair, write $\lambda = Re^{\pm i\theta}$ with $R = \sqrt{\alpha^2 + \beta^2}$ and $\theta = \text{atan2}(\beta, \alpha)$. A standard result on linear recurrences is that the contribution of this pair to (19) can be written as

$$R^n (A \cos(n\theta) + B \sin(n\theta))$$

for suitable real constants A, B determined by the initial conditions. Thus, (19) is always a

linear combination of exponential and sinusoidal terms.

Indifference

We now suppose the person make choices over both lotteries (L_A, L_B) and (G_A, G_B) at wealth level w . Suppose that a person is indifferent between L_A and L_B for wealth levels $w \in \{0, 1, \dots, k_G\}$ and between G_A and G_B for wealth levels $w \in \{0, 1, \dots, k_L\}$. That is, the person is indifferent for choice L over an interval of wealth levels of the length of choice G . These indifferences combine to create $k_G + 1 + k_L + 1$ indifference relations.

As the α -intervals has no intersection by assumption, there are no shared real roots except the simple root $\lambda = 1$. Similarly, we assumed no shared complex roots ($i(L, G) = \emptyset$). Therefore, the non-unit roots for L can be written as $\lambda_1^L, \dots, \lambda_{k_L-1}^L$ and for G be $\lambda_1^G, \dots, \lambda_{k_G-1}^G$, with $\{\lambda_i^L\} \cap \{\lambda_j^G\} = \emptyset$.

From above, u_n for $n \leq k_G + k_L$ must be able to be written as both:

$$u_n = d_0^L + d_1^L(\lambda_1^L)^n + d_2^L(\lambda_2^L)^n + \dots + d_{k_L}^L(\lambda_{k_L}^L)^n \quad (20)$$

$$u_n = d_0^G + d_1^G(\lambda_1^G)^n + d_2^G(\lambda_2^G)^n + \dots + d_{k_G}^G(\lambda_{k_G}^G)^n \quad (21)$$

for some constants $d_0^L, d_1^L, d_2^L, \dots, d_{k_L}^L$ and some constants $\lambda_0^G, d_1^G, d_2^G, \dots, d_{k_G}^G$. Note that, as the choices implied by utility functions are not impacted by constant terms, we can set $\lambda_0^L = \lambda_0^G = 0$ without loss of generality.

Define $r \equiv k_L + k_G$, the number of distinct roots in the union. In matrix form, taking $n = 0, 1, \dots, r - 1$, equation (20) and (21) imply:

$$V_{0,r-1}^L \cdot \vec{d}^L - V_{0,r-1}^G \cdot \vec{d}^G = \vec{u}_{0,r-1} - \vec{u}_{0,r-1} = \vec{0}. \quad (22)$$

Concatenating matrices yields:

$$\begin{bmatrix} V_{0,r-1}^L & V_{0,r-1}^G \end{bmatrix} \begin{bmatrix} \vec{d}^L \\ -\vec{d}^G \end{bmatrix} = \vec{0}, \quad (23)$$

which, when expanded, equals:

$$\left[\begin{array}{cccc|cccc} 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 \\ \lambda_1^L & \lambda_2^L & \cdots & \lambda_{k_L}^L & \lambda_1^G & \lambda_2^G & \cdots & \lambda_{k_G}^G \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ (\lambda_1^L)^{r-1} & (\lambda_2^L)^{r-1} & \cdots & (\lambda_{k_L}^L)^{r-1} & (\lambda_1^G)^{r-1} & (\lambda_2^G)^{r-1} & \cdots & (\lambda_{k_G}^G)^{r-1} \end{array} \right] \begin{bmatrix} d_1^L \\ \vdots \\ d_{k_L}^L \\ -d_1^G \\ \vdots \\ -d_{k_G}^G \end{bmatrix} = \vec{0}. \quad (24)$$

Since there are no shared roots, all r nodes $\{\lambda_1^L, \dots, \lambda_{k_L}^L, \lambda_1^G, \dots, \lambda_{k_G}^G\}$ are distinct. Hence the r -by- r concatenated Vandermonde matrix above is nonsingular, so the only solution to (24) is $d_1^L = \dots = d_{k_L}^L = 0$ and $d_1^G = \dots = d_{k_G}^G = 0$. Therefore $u_n = 0$ for $n = 0, 1, \dots, r-1$; i.e., the only utility sequence that satisfies both sets of indifference relations on this interval is the trivial (zero) sequence.

Note that the proof actually only required that u_n be written in the forms in (20) and (21) for all $n \leq r-1 = k_L + k_G - 1$ rather than for all $n \leq k_G + k_L$. Therefore, we could have assumed indifference between L_A and L_B for wealth levels $w \in \{0, 1, \dots, k_G - 1\}$ and between G_A and G_B for wealth levels $w \in \{0, 1, \dots, k_L - 1\}$ and the same proof would hold.

Also, note that there are necessarily non-null solutions not using the shared roots if the indifference intervals change to $\{0, 1, \dots, k_G - 2\}$ and $\{0, 1, \dots, k_L - 2\}$, respectively. In this case, we would be solving for d given an $(r-1)$ -by- (r) matrix, which must have infinite solutions. Therefore, there does exist a rationalizing utility function in this case (although it does not need to be increasing).

D Proof: Weak Preference Case

D.1 Definitions

D.1.1 Utility Gaps

Define differences in utility at w as

$$\Delta_w \equiv u(w) - u(w-1), \quad \text{where } \Delta_w > 0 \text{ for all } w$$

Next, define the *cumulative sum of* $b_i = p_i^A - p_i^B$ as:

$$c_i \equiv \sum_{j=i}^k b_j$$

Whereas d is a $k + 1$ -length vector, c is an k -length vector with indexes 1 to k .

Define the *utility gap* of L_A vs L_B if we start with wealth w as:

$$g_w \equiv E[u, L_A; w] - E[u, L_B; w]$$

Rearranging the sums and collecting terms on each Δ_n yields ($c_0 = 0$ such that the $i = 0$ term drops out):

$$u_{w+i} = u_w + \sum_{j=1}^i \Delta_{w+j}$$

But then:

$$\begin{aligned} g_w &\equiv E[u, L_A; w] - E[u, L_B; w] \\ &= \sum_{i=0}^k b_i u_{w+i} \\ &= \sum_{i=0}^k b_i \left(u_w + \sum_{j=1}^i \Delta_{w+j} \right) \\ &= \sum_{i=0}^k b_i u_w + \sum_{i=1}^k \sum_{j=1}^i b_i \Delta_{w+j} \\ &= \sum_{i=0}^k \sum_{j=1}^i b_i \Delta_{w+j} \\ &= \sum_{i=1}^k c_i \Delta_{w+i} \end{aligned}$$

So

$$g_w \equiv \sum_{i=1}^k c_i \Delta_{w+i}$$

That is, the gap in utility from lottery A and lottery B can be seen as a weighted average of Δ_{w+i} terms with weights c_i (or, as a weighted average of c_i terms with weights Δ_{w+i}).

D.1.2 Reference Utility Function and the Gap Polynomial

The theorem concerns α -intervals, which invoke the exponential utility function $r(x; \alpha) = -e^{-\alpha x}/\alpha$ (with $r(x; 0) = x$ as a reference utility function. For notational ease in the proofs, we derive a simplified expression for the exponential utility gap. Let $t = e^{-\alpha}$. The gap under exponential utility is:

$$g_w^{ref}(\alpha) \equiv E[r, L_A; w] - E[r, L_B; w] = \frac{1-t}{\alpha} t^{w-1} P(t),$$

where $P(t) \equiv \sum_{i=1}^k c_i t^i$ is the *gap polynomial*.²³

Thus, $\text{sign}(g_w^{ref}(\alpha)) = \text{sign}(P(t))$, and the exponential yields the same rank orderings as the sign of $P(t)$, and indifferences for exponential utility occur at positive roots of $P(t) = 0$. Therefore, without loss of generality for preferences, we work with the simplified gap polynomial $P(t)$ in the proofs, as it preserves the exponential rankings exactly (note also that at $t = 1$ corresponding to $(\alpha = 0)$, $P(1) = \mu[L_A] - \mu[L_B]$, matching the risk-neutral case).

D.1.3 Summing over gaps

Recall that our statements rely on the fact that a person makes consistent decisions over different wealth levels. To exploit this fact, we aggregate these gaps over a wealth horizon of $\delta > k$ by taking a weighted sum of these decisions using exponential weights of $\lambda > 0$:

$$G_{w, w+\delta}(\lambda) \equiv \sum_{i=0}^{\delta} \lambda^i g_{w+i}.$$

Crucially, if $g_{w+i} \geq 0$ for all wealth levels between w and $w + \delta$, then $G \geq 0$. The sign of G therefore encodes the consistency of decisions across wealth levels. Collecting like Δ -terms yields

$$G_{w, w+\delta}(\lambda) = \underbrace{\sum_{i=1}^{k-1} \lambda^i L_i(\lambda) \Delta_{w+i}}_{k-1 \text{ Leading Terms}} + \underbrace{\sum_{i=k}^{\delta+1} \lambda^i M(\lambda) \Delta_{w+i}}_{\text{Middle Terms}} + \underbrace{\sum_{i=2}^k \lambda^{\delta+i} T_i(\lambda) \Delta_{w+\delta+i}}_{k-1 \text{ Trailing Terms}},$$

where

$$L_i(\lambda) \equiv \sum_{j=1}^i c_j (1/\lambda)^j, \quad M(\lambda) \equiv \sum_{j=1}^k c_j (1/\lambda)^j, \quad T_i(\lambda) \equiv \sum_{j=i}^k c_j (1/\lambda)^j.$$

²³Note that $\frac{1-t}{\alpha} > 0$ for all $t > 0$, $t \neq 1$ (with limit 1 as $t \rightarrow 1$), and $t^{w-1} > 0$ always. If $\alpha > 0$ ($t < 1$), then $1-t > 0$ and $\alpha > 0$, and therefore positive. If $\alpha < 0$ ($t > 1$), then $1-t < 0$ and $\alpha < 0$, and therefore positive. As $\alpha \rightarrow 0$ ($t \rightarrow 1$), the limit is $1 > 0$ and therefore positive.

To see how $G_{w,w+\delta}(\lambda)$ is formed, consider the explicit expansion of $G_{w,w+6}(\lambda) = \sum_{i=0}^6 \lambda^i g_{w+i}$ at $w = 0$:

	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	Δ_6	Δ_7	Δ_8	Δ_9
i=0	c_1	c_2	c_3						
i=1		$c_1\lambda$	$c_2\lambda$	$c_3\lambda$					
i=2			$c_1\lambda^2$	$c_2\lambda^2$	$c_3\lambda^2$				
i=3				$c_1\lambda^3$	$c_2\lambda^3$	$c_3\lambda^3$			
i=4					$c_1\lambda^4$	$c_2\lambda^4$	$c_3\lambda^4$		
i=5						$c_1\lambda^5$	$c_2\lambda^5$	$c_3\lambda^5$	
i=6							$c_1\lambda^6$	$c_2\lambda^6$	$c_3\lambda^6$

So, for example, there are two leading terms with $c_1\Delta_1$ and $(c_2 + c_1\lambda)\Delta_2$, there are 5 middle terms with a power of λ times $(c_3 + c_2\lambda + c_1\lambda^2)$ and a Δ term, and two trailing terms with a power of λ times $c_3 + c_2\lambda\Delta_8$ and $c_3\Delta_9$.

Note that:

$$M(\lambda) \equiv \sum_{j=1}^k c_j (1/\lambda)^j = P(1/\lambda),$$

where $P(t) = \sum_{i=1}^k c_i t^i$ is the previously-defined gap-polynomial. That is, $M(\lambda)$ has the same sign as the reference gap utility of a person with exponential utility parameterized by $t = 1/\lambda$. As $\Delta_w > 0$, the sign of the middle terms is determined entirely by whether a reference exponential person with $t = 1/\lambda$ would choose L_A over L_B . For example, when $\lambda = 1$, $M(1)$ equals the expected value gap between the two lotteries, and determines the choice of an exponential person with $t = 1$ (who is risk-neutral).

$\sum_{i=1}^{k-1} \lambda^i L_i(\lambda) \Delta_{w+i}$ can be seen as representing a “virtual” $k - 1$ length decision with c_i values equal to $\lambda^i L_i(\lambda)$, and represented by gap-polynomial $P^L(\lambda)$. Intuitively, this decision overweights earlier components of c_i relative to the original choice. Similarly, $\sum_{i=2}^k \lambda^{\delta+i} T_i(\lambda) \Delta_{w+\delta+i}$ can be seen as a virtual $k - 1$ length choice that overweights later components, represented by gap-polynomial $P^T(\lambda)$. The proofs rely heavily on the behavior for these virtual decisions.

D.1.4 Tilted Choices and Utilities

In this section, we define the γ -tilted utility gaps as

$$\Delta_{w+i}^{(\gamma)} = \gamma^i \Delta_{w+i} \quad \forall w, i,$$

and constructing γ -tilted utility $u^{(\gamma)}$

$$u^{(\gamma)} = \sum_{w=0}^{\infty} \Delta_w^{(\lambda)}$$

and the γ -tilted lotteries $L_A^{(\gamma)}$ and $L_B^{(\gamma)}$ by re-weighting each probability $p_{\cdot,i}$ to

$$p_{\cdot,i}^{(\gamma)} = \frac{\gamma^{-i} p_{\cdot,i}}{\sum_{j=1}^k \gamma^{-j} p_{\cdot,j}},$$

so that the new weights satisfy $b_i^{(\gamma)} \propto \gamma^{-i} b_i$ and hence

$$c_i^{(\gamma)} = \sum_{j=i}^k b_j^{(\gamma)} \propto \gamma^{-i} c_i.$$

By construction, the sign of the utility gap is then invariant under tilting as:

$$g_w^{(\gamma)} = \sum_{i=1}^k c_i^{(\gamma)} \Delta_{w+i}^{(\gamma)} \propto \sum_{i=1}^k (\gamma^{-i} c_i) (\gamma^i \Delta_{w+i}) = \sum_{i=1}^k c_i \Delta_{w+i} = g_w.$$

Consequently, the t -interval of the tilted choice is obtained simply by rescaling the argument in the gap-polynomial. The tilted gap-polynomial becomes

$$P^{(\gamma)}(t) = \sum_{i=1}^k c_i^{(\gamma)} t^i \propto \sum_{i=1}^k \gamma^{-i} c_i t^i = P(t/\gamma).$$

It follows that each root r of the original P is carried to $r\gamma$ in $P^{(\gamma)}$. We state an immediate Lemma for use later:

Lemma 1. *Suppose that there exists a utility function with Δ_w such that L_B is preferred over L_A . Then, there exists a utility function $\Delta_w^{(\lambda)}$ such that $L_B^{(\lambda)}$ is preferred over $L_A^{(\lambda)}$.*

—*Proof of Lemma 1.* Given the definitions above the fact that the gaps g_w and $g_w^{(\lambda)}$ have the same sign, the statement follows immediately

—*End of Proof of Lemma 1.* □

D.2 Building Intuition: Simple Risk-Seeking vs. Risk-Averse Bets

We start with “simple” bets for intuition on the proof strategy, although this proof is subsumed by the later more general proof of “complex” bets.

Simple lotteries occur when the cumulative weights c_i pass zero exactly once. In this case, Descartes’ Rule of Signs implies there is exactly one real root in the gap-polynomial

$P(t)$. This is equivalent to a t -interval from a weak-preference decision of the form $(0, x]$ or $[x, \infty)$ (which is equivalent to an α -interval of the form $[\ln(x), \infty)$ or $(-\infty, \ln(x)]$). Simple lotteries represent most lotteries studied in the theoretical and experimental literature, and can (loosely) be seen as a tradeoff between mean and spread.

The parameter c_1 (which has the opposite sign of b_1) plays an important role in the form of the associated t -interval as it determines whether the cumulative weights c_i start negative and move positive or vice versa. Intuitively, if lottery A places more weight on the lowest possible outcome, then $b_1 > 0$ and $c_1 < 0$, and the most extreme risk-averse person will choose A . In this case then, the t -interval can only take the form $(0, x]$ for a preference for lottery A . Alternatively, if $b_1 < 0$ and $c_1 > 0$, then the t -interval can only take the form $[x, \infty)$.

D.2.1 Risk-averse choices: Δ_w must limit to 0

We first consider simple “risk-averse” choices with t -intervals of the form $(0, x]$ where $x < 1$. These are choices that only some exponential risk-averse agents will take, but a risk-neutral and all risk-seeking agents will not choose. An example of a simple risk-averse choice is weakly preferring \$1 to a 50% chance of \$3.

Lemma 2 (Consistent Simple Risk-Averse Choice Implies Bounded Utility Sum and $\Delta_w \rightarrow 0$). *Suppose an person has a weak preference for the simple risk-averse choice L_B over L_A for all wealth levels $w \geq w_0$. Then, $\sum_{i=1}^{\infty} \Delta_{w+i} < \infty$ and $\Delta_{w+i} \rightarrow 0$ as $i \rightarrow \infty$.*

—*Proof of Lemma 2.* For this section, we use $\lambda = 1$ to construct the gap sum $G_{w,w+\delta}(1)$. That is, G represents the straightforward sum of all of the utility gaps from w to $w + \delta$. Now, consider a person who makes a risk-averse choice B from wealth levels w to $w + \delta$. As expected value favors decision A , $M(1) = \sum_{j=1}^k c_j > 0$. As the most risk-averse person favors B , $c_1 < 0$. Therefore, as c_i starts negative and passes zero once (becoming positive), $T_i(1) = \sum_{j=i}^k c_j > 0$ for all i (partial sums from the positive tail dominate). Additionally, the leading coefficients $L_i(1) = \sum_{j=1}^i c_j < 0$ (early negative c_j dominate the partial sums). Combining this with $G_{w,w+\delta}(1) \leq 0$ (by the person’s consistent choice of B , implying $g_{w+i} \leq 0$ for $i = 0$ to δ):

$$\begin{aligned} 0 &\geq G_{w,w+\delta}(1) = \sum_{i=1}^{k-1} L_i(1) \Delta_{w+i} + \sum_{i=k}^{\delta+1} M(1) \Delta_{w+i} + \sum_{i=2}^k T_i(1) \Delta_{w+\delta+i} \\ &> \sum_{i=1}^{k-1} L_i(1) \Delta_{w+i} + \sum_{i=k}^{\delta+1} M(1) \Delta_{w+i}, \end{aligned}$$

where the strict inequality follows from dropping the positive trailing terms ($T_i(1) > 0$,

$\Delta > 0$). Rearranging gives:

$$\sum_{i=k}^{\delta+1} M(1) \Delta_{w+i} < - \sum_{i=1}^{k-1} L_i(1) \Delta_{w+i}.$$

Dividing both sides by $M(1) > 0$ (which preserves the inequality):

$$\sum_{i=k}^{\delta+1} \Delta_{w+i} < \sum_{i=1}^{k-1} -\frac{L_i(1)}{M(1)} \Delta_{w+i}.$$

The right-hand side is a fixed (δ -independent) finite sum. Since $L_i(1) < 0$ and $M(1) > 0$, each coefficient $-L_i(1)/M(1) > 0$, so the right-hand side is positive. However, as $\delta \rightarrow \infty$, the left-hand side is a sum of increasingly many positive terms $\Delta_{w+i} > 0$. For the inequality to hold for arbitrarily large δ , we must have $\sum_{i=k}^{\delta+1} \Delta_{w+i} < \infty$ and therefore $\Delta_{w+i} \rightarrow 0$ as $i \rightarrow \infty$ (otherwise, the left-hand side would grow without bound, eventually exceeding the fixed right-hand side).

—End of Proof of Lemma 2. □

D.2.2 Risk-seeking choices: Δ_w cannot limit to 0

Now, we consider “risk-seeking” choices with t -intervals of the form $[x, \infty)$ where $x > 1$. These are choices that only some exponential risk-seeking agents will take, but a risk-neutral and all risk-averse agents will not choose. An example of a simple risk-seeking choice is weakly preferring a 50% chance of \$10 to \$6.

Lemma 3 (Consistent Simple Risk-Seeking Choice Implies $\Delta_w \not\rightarrow 0$). *Suppose an person has a weak preference for the simple risk-seeking choice L_B over L_A for all wealth levels $w \geq w_0$. Then, $\limsup_{i \rightarrow \infty} \Delta_{w+i} = \infty$.*

—Proof of Lemma 3. Now, consider a person who makes a risk-seeking choice B from wealth levels w to $w+\delta$. As expected value favors decision A , $M(1) = \sum_{j=1}^k c_j > 0$. As the most risk-averse person favors A , $c_1 > 0$. Therefore, as c_i starts positive and passes zero once (becoming negative), $L_i(1) = \sum_{j=1}^i c_j > 0$ for all i (positive terms dominate partial sums). Additionally, the trailing coefficients $T_i(1) = \sum_{j=i}^k c_j < 0$ for all $i \geq 2$ (negative tail dominates). Following similar logic as in the risk-averse case with $G_{w,w+\delta}(1) \leq 0$, $M(1) > 0$, and $L_i(1) > 0$, we drop the positive leading terms to obtain:

$$0 \geq G_{w,w+\delta}(1) > \sum_{i=k}^{\delta+1} M(1) \Delta_{w+i} + \sum_{i=2}^k T_i(1) \Delta_{w+\delta+i}.$$

Rearranging gives:

$$\sum_{i=k}^{\delta+1} \Delta_{w+i} < \sum_{i=2}^k -\frac{T_i(1)}{M(1)} \Delta_{w+\delta+i}.$$

Since $T_i(1) < 0$ and $M(1) > 0$, each coefficient $-T_i(1)/M(1) > 0$, so the right-hand side is positive.

Now, suppose that $\Delta_{w+i} \rightarrow 0$ as $i \rightarrow \infty$. Then, as $\delta \rightarrow \infty$, the right-hand side (a finite sum of terms that approach zero) must approach 0. However, the left-hand side is a sum of positive terms over an increasing number of indices. In particular, for any fixed $N \geq k$, the left-hand side is some positive constant that is greater than $\sum_{i=k}^N \Delta_{w+i} > 0$. But, then for a large enough δ , the right-hand side must drop below that constant. This is a contradiction and therefore $\Delta_{w+i} \not\rightarrow 0$.

Next, suppose that the sequence Δ_{w+i} is bounded above by some $U > 0$ as $i \rightarrow \infty$. Then, the right-hand side is bounded above by $\left(\max_{i=2}^k -T_i(1)/M(1)\right) \cdot (k-1) \cdot M$, a finite constant. However, for the left-hand side, we showed above that $\Delta_{w+i} \not\rightarrow 0$. Therefore, as $\delta \rightarrow \infty$, the left-hand side $\rightarrow \infty$. This violates the inequality, leading to a contradiction. Thus, Δ_{w+i} cannot be bounded above. Combined, Δ_{w+i} cannot limit to 0, and the partial sums $\sum_{i=1}^{\delta} \Delta_{w+i}$ must be unbounded as $\delta \rightarrow \infty$.

—End of Proof of Lemma 3. □

D.2.3 Inconsistent Simple Choices

Combining the above statements suggests that a person cannot consistently weakly prefer the risk-averse” simple choice L_B over L_A (implying that $\Delta_w \rightarrow 0$) while also consistently weakly preferring the risk-seeking” simple choice G_B over G_A (implying that $\Delta_w \not\rightarrow 0$). This violation only requires that the person is an EUM with increasing utility. Stated formally:

Appendix-Proposition 1. *An EUM cannot weakly prefer a simple risk-seeking choice L_B over L_A and a simple risk-averse choice G_B over G_A for all wealth levels.*

—Proof of Appendix-Proposition 1. By the risk-averse section, consistent preference for L_B over L_A implies $\Delta_w \rightarrow 0$ as $w \rightarrow \infty$ by Lemma 2. By the risk-seeking section, consistent preference for G_B over G_A implies $\Delta_w \not\rightarrow 0$ as $w \rightarrow \infty$ by 3. This is a contradiction for any increasing utility ($\Delta_w > 0$).

—End of Proof of Appendix-Proposition 1. □

D.2.4 Expanding Incompatible Choices Using Tilted Utilities

The statement above shows that choices with t -intervals of $(0, x]$ and $[y, \infty)$ where $x < 1 < y$ are incompatible. That is, a person cannot simultaneously consistently take “risk-averse”

and “risk-seeking” choices. Or, put differently, a person cannot simultaneously consistently take “more-risk-averse-than-($t=1$)” and “more-risk-seeking-than-($t=1$)” choices. We aim to expand the statement above to all simple choices, where ($x < z < y$). Intuitively, we desire to show that a person cannot simultaneously consistently take “more-risk-averse-than-($t=z$)” and “more-risk-seeking-than-($t=z$)” choices.

The use of tilted utilities allows us to make this connection. Specifically, if a person has a utility function that justifies taking “below- z ” and “above- z ” choices consistently, then there exists a tilted utility function that justifies taking “below-1” and “above-1” choices consistently. But, from Proposition 1, we know that this cannot be true. To formalize this, define, $t(L_B \succeq L_A) = \{t | P(t) \leq 0\}$ and $t(G_B \succeq B_A) = \{t | P(t) \leq 0\}$ recalling that $P(t) = \sum_{i=1}^k c_i t^i$, $c_i = \sum_{j=i}^k b_j$, and $b_j = p_j^A - p_j^B$ for all lotteries X_A, X_B .

Appendix-Proposition 2. *An EUM cannot choose simple choice $L_B \succeq L_A$ and simple choice $G_B \succeq G_A$ for all wealth levels if $\inf\{t(L_B \succeq L_A)\} > \sup\{t(G_B \succeq G_A)\}$.*

—*Proof of Appendix-Proposition 2.* Note that by the definition of simple choice, $t(L_B \succeq L_A) = [y, \infty)$ and $t(G_B \succeq G_A) = (0, x]$ where $x < y$. Now, suppose that an utility function exists that rationalizes both decisions. Choose some z such that $x < z < y$. Then, by Lemma 1, there is a $1/z$ -tilted utility function that rationalizes both $1/z$ -tilted decisions. The tilted first choice will have t -interval $(0, x/z]$ where $x/z < z/z = 1$, and therefore by Lemma 2, the tilted utility function $\Delta_w^{(\lambda=z)} \rightarrow 0$. The tilted second choice will have t -interval $[y/z, \infty)$ where $y/z > z/z = 1$, and therefore by Lemma 3, the tilted utility function $\Delta_w^{(\lambda=z)} \not\rightarrow 0$. Thus, this tilted utility function cannot exist, leading to a contradiction. Thus, the original utility function cannot exist.

—*End of Proof of Appendix-Proposition 2.* □

D.3 Complex Choices

We now extend the proof to “complex” choices, in which c_i does not cross zero only once (such that $P(t)$ may have zero or more than one root).

D.3.1 No-Root Choices: Incompatibility

We start with the case in which t -intervals of choice B is null. In this case, we show that an EUM cannot consistently take this choice (and, in this case, there is a computable bound on the wealth interval provided by previous work in mathematics):

Proposition 3. *An EUM cannot weakly prefer choice L_B over L_A for wealth levels $w \in \{0, 1, \dots, N\}$ if the t -interval of choice B is $\emptyset$, where N is the bound provided in Powers*

and Reznick (2001) or Castle et al. (2011).

—*Proof of Proposition 3.* The proof uses a binomially-weighted sum of gaps and then invokes Pólya’s theorem. As this is likely not familiar to economists, we restate it here:

Statement: the one-variable case of Pólya’s Theorem: If a real polynomial $P(t) > 0$ for all $t > 0$, then there exists a finite integer N^* such that every coefficient of $(1+t)^{N^*} P(t)$ is weakly positive (and one is strictly positive).

[Later work provides explicit bounds for N^* given the coefficients of $P(t)$. Powers and Reznick (2001) give an explicit computable bound for N^* and Castle et al. (2011) provide a bound given no constant term in $P(t)$.]

Intuitively, we are going to start by following the same strategy as above in noting that a preference for B implies $g_w \leq 0$ for $w \in [0, N^*]$, which implies that a positively weighted sum of g_w terms must be ≤ 0 . Guided by Pólya, we are going to choose weights such that the coefficients in the sum match the coefficients of $(1+t)^{N^*} P(t)$. But, by the Pólya theorem, the coefficients are all positive. But, then, the sum (of positive coefficients times positive Δ_w terms) must be positive, and we have a contradiction.

To start, we construct the sum by fixing an integer $N \geq 0$ and forming:

$$S_w^{(N)} := \sum_{i=0}^N \binom{N}{i} g_{w+i}.$$

Intuitively, this is a *finite* nonnegative-weighted average of nearby wealth comparisons. As $g_{w+i} < 0$, $S_w^{(N)}$ must be negative. Our goal is to show this leads to a contradiction.

Substitute $g_{w+i} = \sum_{j=1}^k c_j \Delta_{w+i+j}$ into $S_w^{(N)}$ and group like terms:

$$S_w^{(N)} = \sum_{i=0}^N \binom{N}{i} \sum_{j=1}^k c_j \Delta_{w+i+j} = \sum_{s=1}^{N+k} \left(\sum_{j=1}^k c_j \binom{N}{s-j} \right) \Delta_{w+s}.$$

The s th coefficient on Δ_{w+s} is:

$$b_s(N) := \sum_{j=1}^k c_j \binom{N}{s-j} \quad (s = 1, \dots, N+k),$$

such that:

$$S_w^{(N)} = \sum_{s=1}^{N+k} b_s(N) \Delta_{w+s}.$$

As a simple example, if $N = 2$ (weights 1, 2, 1), a one-line expansion gives

$$S_w^{(2)} = \underbrace{c_1}_{b_1} \Delta_{w+1} + \underbrace{(2c_1 + c_2)}_{b_2} \Delta_{w+2} + \underbrace{(c_1 + 2c_2 + c_3)}_{b_3} \Delta_{w+3} + \underbrace{(c_2 + 2c_3)}_{b_4} \Delta_{w+4} + \underbrace{c_3}_{b_5} \Delta_{w+5}.$$

The next step is to show that the $b_s(N)$ terms are equal to the coefficients of $(1+t)^N P(t)$:

$$(1+t)^N P(t) = \left(\sum_{m=0}^N \binom{N}{m} t^m \right) \left(\sum_{j=1}^k c_j t^j \right) = \sum_{s=1}^{N+k} \left(\sum_{j=1}^k c_j \binom{N}{s-j} \right) t^s = \sum_{s=1}^{N+k} b_s(N) t^s.$$

But, then the Pólya theorem says that there exists $N = N^*$ so all $b_s(N^*) > 0$. But, then $S_w^{(N^*)} > 0$ as $b_s(N^*) > 0$ and $\Delta_{w+s} > 0$. This is a contradiction. Therefore, we conclude that the person cannot be an EUM.

—End of Proof of Proposition 3. □

D.3.2 Generalized Risk-Seeking Choices: Δ_w Cannot Limit to 0

We define a generalized risk-seeking choice as one in which the t -intervals of choice B does not include any part of $(0, 1]$ and includes some subset of $(1, \infty)$. In other words, $P(t) > 0$ for all $(0, 1]$ and the roots of $P(t)$ must all be greater than 1. We label these roots as $1 < r_1 < r_2 < \dots < r_m$. Note that by Descartes' Rule of Signs there must be fewer than k roots.

Lemma 4 (Consistent Complex Risk-Seeking Choice Implies $\Delta_w \not\rightarrow 0$). *Suppose an person has a weak preference for the complex risk-seeking choice L_B over L_A for all wealth levels $w \geq w_0$. Then, $\sum_{i=1}^{\infty} \Delta_{w+i} = \infty$ and $\Delta_{w+i} \not\rightarrow 0$ as $i \rightarrow \infty$.*

—Proof of Lemma 4. Consider an agent who takes B over many wealth levels. Then $0 \geq g_w$ for any w and therefore $0 \geq G_{w,w+\delta}(\lambda) \equiv \sum_{i=0}^{\delta} \lambda^i g_{w+i}$ for any δ and λ . Therefore

$$\sum_{i=1}^{k-1} \lambda^i L_i(\lambda) \Delta_{w+i} \leq - \sum_{i=k}^{\delta+1} \lambda^i M(\lambda) \Delta_{w+i} - \sum_{i=2}^k \lambda^{\delta+i} T_i(\lambda) \Delta_{w+\delta+i}.$$

Now, we set $\lambda = 1/r_1 \in (0, 1)$. We suppose for the sake of contradiction that $\Delta_w \rightarrow 0$ as $w \rightarrow \infty$. Consider the first term on the right-hand side. Recall that $M(\lambda) \equiv \sum_{j=1}^k c_j (1/\lambda)^j = P(1/\lambda)$. But, as $\lambda = 1/r_1$, then $P(1/\lambda) = P(r) = 0$. Therefore the middle term is zero. Next, consider the second term on the right-hand side. As $\lambda \in (0, 1)$, then as $\delta \rightarrow \infty$, the term $\sum_{i=2}^k \lambda^{\delta+i} T_i(\lambda) \Delta_{w+\delta+i}$ tends to 0 (since $\lambda^{\delta} \rightarrow 0$ and $\Delta_{w+\delta+i} \rightarrow 0$).

Consequently, the right-hand side tends to 0 as δ rises. Taking the limit as $\delta \rightarrow \infty$, we

have

$$\sum_{i=1}^{k-1} \lambda^i L_i(\lambda) \Delta_{w+i} \leq 0.$$

And, as this argument holds at all wealth levels, this inequality must be true for any w . One way to interpret this inequality is that a person who chooses B at all wealth levels for lotteries represented by c_i must also choose B at all wealth levels given the virtual lotteries represented by cumulants $\lambda^i L_i(\lambda)$.

What do we know about the choice represented by $\lambda^i L_i(\lambda)$? In particular, what are its t -intervals? The gap polynomial for this choice is

$$P^L(t; \lambda) = \sum_{i=1}^{k-1} \lambda^i L_i(\lambda) t^i$$

First, we note that:

Sub-Lemma 4.1. $(1 - \lambda t) P^L(t; \lambda) = P(t) - (\lambda t)^k P\left(\frac{1}{\lambda}\right).$

—*Proof of Sub-Lemma 4.1.* Expand P^L by swapping the order of summation (and setting $m := j - i$):

$$P^L(t; \lambda) = \sum_{j=1}^{k-1} t^j \sum_{i=1}^j \lambda^{j-i} c_i = \sum_{i=1}^{k-1} c_i t^i \sum_{m=0}^{k-1-i} (\lambda t)^m.$$

Then, we note the geometric-series identity:

$$\sum_{m=0}^{k-1-i} (\lambda t)^m = \frac{1 - (\lambda t)^{k-i}}{1 - \lambda t}.$$

Substituting and regrouping:

$$P^L(t; \lambda) = \frac{1}{1 - \lambda t} \left[\underbrace{\sum_{i=1}^{k-1} c_i t^i}_{P(t) - c_k t^k} - (\lambda t)^k \underbrace{\sum_{i=1}^{k-1} c_i \lambda^{-i}}_{P\left(\frac{1}{\lambda}\right) - c_k} \right].$$

The $c_k t^k$ terms cancel, leaving

$$(1 - \lambda t) P^L(t; \lambda) = P(t) - (\lambda t)^k P\left(\frac{1}{\lambda}\right),$$

—*End of Proof of Sub-Lemma 4.1.* □

Given this form, $P^L(t; \lambda)$ has useful properties:

Sub-Lemma 4.2. *Suppose the roots of $P(t)$ are $r_1, \dots, r_m$ with $m < k$. Fix $\lambda = 1/r_m$. Then*

1. The roots of $P^L(t; \lambda)$ are $r_1, r_2, \dots, r_{m-1}$
2. For any $t < r_m$, $\text{sign}(P^L(t; \lambda)) = \text{sign}(P(t))$

—*Proof of Sub-Lemma 4.2.* Because $P(r_i) = 0$, the identity in Lemma (4.1) with $\lambda = 1/r_m$ simplifies to

$$(1 - \lambda t) P^L(t; \lambda) = P(t). \quad (*)$$

First, consider any $0 < t < r_m$. Then, $t\lambda = t/r_m < 1$, such that $1 - t\lambda > 0$. Therefore, $\text{sign}(P^L(t; \lambda)) = \text{sign}(P(t))$ proving (2). Similarly, $P^L(t; \lambda) = 0$ only if $P(t) = 0$, proving that $r_1, \dots, r_{m-1}$ are the only roots of $P^L(t; \lambda)$ for $t < r_m$. Also, if $t > r_m$, then $1 - t\lambda < 0$, so that $P^L(t; \lambda) = 0$ only if $P(t) = 0$, such that there are no roots if $t > r_m$.

Finally, consider if $t = r_m$. Factor $P(t) = (t - 1/\lambda)Q(t)$ with $Q(1/\lambda) \neq 0$ (simple root). Then (*) becomes:

$$P^L(t; \lambda) = \frac{(t - 1/\lambda)Q(t)}{1 - t\lambda}.$$

Numerator and denominator share the same simple zero at $t = 1/\lambda$, so their ratio has a finite, non-zero limit there; consequently $1/\lambda$ is *not* a root of P^L .

—*End of Proof of Sub-Lemma 4.2.* □

It is useful to break and consider where we are. We started with the person consistently taking choice B given lottery represented by c_i , where B is never taken by a risk-averse exponential agent but may be taken by a risk-seeking exponential agent. We proved that, if $\Delta_w \rightarrow 0$, this person must also consistently take choice B given the lottery represented by $\lambda^i L_i(\lambda)$, where B is never taken by a risk-averse exponential agent but may be taken by a risk-seeking exponential agent.

Now, suppose that the lottery represented by $\lambda^i L_i(\lambda)$ has all positive coefficients and therefore $g_w > 0$ for all w , contradicting the requirement $g_w \leq 0$. We then have a contradiction and conclude that our assumption that Δ_w goes to zero must be false.

However, this may not be true. But, given that the new lottery has the same characteristics as the old one but with one fewer root, we can repeat the argument. That is, we can set the new c_i to the coefficients of $P^L(t; \lambda)$ (rescaled by constants if necessary, as scaling does not affect intervals). However, as we iterate, the Lemma shows that we lose one root each time. Therefore, after some finite number of iterations, we either must end with all positive coefficients (which leads to a contradiction as above), or to the removal of all roots.

Consider the case in which the iteration leads to a virtual decision with no roots. It must be that for this decision $P(t) > 0$ for $t \in (0, 1]$. Given there are no roots, then $P(t) > 0$ for all $t > 0$, such that the t -partition of choice B is $\emptyset$. In this decision, the person consistently weakly prefers L_B over L_A . But, this is not possible by Proposition 3 and so we have a contradiction. Therefore $\Delta_w \not\rightarrow 0$ as $w \rightarrow \infty$.

—End of Proof of Lemma 4. □

D.3.3 Generalized Risk-Averse Choices: Δ_w must limit to 0

We now move to generalized risk-averse choices, which follows a similar but somewhat complementary tack, in which we focus on iterating over the trailing term rather than the leading term. In these decisions, the t -intervals of choice B does not include any part of $[1, \infty)$ and includes some subset of $(0, 1)$. In other words, $P(t) > 0$ for all $t \in [1, \infty)$ and the roots of $P(t)$ must all be less than 1. We label these roots as $0 < r_1 < r_2 < \dots < r_m < 1$. Note that by Descartes' Rule of Signs there must be no more than k roots.

Lemma 5 (Consistent Complex Risk-Averse Choice Implies $\Delta_w \rightarrow 0$). *Suppose a person has a weak preference for the complex risk-averse choice L_B over L_A for all wealth levels $w \geq w_0$. Then, $\Delta_{w+i} \rightarrow 0$ as $i \rightarrow \infty$.*

—Proof of Lemma 5. Consider an agent who takes B over many wealth levels. Then $0 \geq g_w$ for any w and therefore $0 \geq G_{w,w+\delta}(\lambda)$ for any δ and λ . Therefore

$$\sum_{i=1}^{k-1} \lambda^i L_i(\lambda) \Delta_{w+i} \leq - \sum_{i=k}^{\delta+1} \lambda^i M(\lambda) \Delta_{w+i} - \sum_{i=2}^k \lambda^{\delta+i} T_i(\lambda) \Delta_{w+\delta+i}.$$

Now, we set $\lambda = 1/r_m > 1$. We suppose for the sake of contradiction that $\Delta_w \not\rightarrow 0$ as $w \rightarrow \infty$. As $M(\lambda) \equiv P(1/\lambda) = P(r_m) = 0$, the middle term is zero. The inequality becomes:

$$\sum_{i=2}^k \lambda^{\delta+i} T_i(\lambda) \Delta_{w+\delta+i} \leq - \sum_{i=1}^{k-1} \lambda^i L_i(\lambda) \Delta_{w+i}.$$

Dividing both sides by $\lambda^\delta > 0$ (preserving inequality direction):

$$\sum_{i=2}^k \lambda^i T_i(\lambda) \Delta_{w+\delta+i} \leq - (1/\lambda^\delta) \sum_{i=1}^{k-1} \lambda^i L_i(\lambda) \Delta_{w+i}.$$

But, now, just as in the risk-seeking case above, the right-hand side (a bounded number multiplied by a factor $(1/\lambda)^\delta = r_m^\delta$) goes to zero as $\delta \rightarrow \infty$. Therefore, as above, there must

be a δ_0 at which, for all $\delta > \delta_0$,

$$\sum_{i=2}^k \lambda^i T_i(\lambda) \Delta_{w+\delta+i} \leq 0.$$

If not, then $\sum_{i=2}^k \lambda^i T_i(\lambda) \Delta_{w+\delta+i} > 0$ infinitely often, the inequality would fail for those δ given small $(1/\lambda^\delta) \sum_{i=1}^{k-1} \lambda^i L_i(\lambda) \Delta_{w+i}$. This implies that starting from wealth $w + \delta_0 + 1$, the utility gap for the new lottery defined by the normalized trailing coefficients is ≤ 0 , meaning the person chooses B consistently for all wealth levels from $w + \delta_0 + 1$ onward on this new lottery.

The gap polynomial for this new choice (up to scaling) is

$$P^T(t; \lambda) = \sum_{i=1}^{k-1} (\lambda t)^i T_{i+1}(\lambda),$$

where the sum is reindexed to $i = 1$ to match the risk-seeking case above. We now iterate on this new lottery, using the following lemmas analogous to the risk-seeking case.

Sub-Lemma 5.1. $(1 - \lambda t) P^T(t; \lambda) = \lambda t P\left(\frac{1}{\lambda}\right) - P(t).$

—*Proof of Sub-Lemma 5.1.* We have

$$P^T(t; \lambda) = \sum_{i=1}^{k-1} (\lambda t)^i \sum_{j=i+1}^k c_j (1/\lambda)^j = \sum_{j=2}^k c_j (1/\lambda)^j \sum_{i=1}^{j-1} (\lambda t)^i.$$

The inner sum is

$$\sum_{i=1}^{j-1} (\lambda t)^i = (\lambda t) \frac{1 - (\lambda t)^{j-1}}{1 - \lambda t} = \frac{(\lambda t) - (\lambda t)^j}{1 - \lambda t}.$$

Thus,

$$P^T(t; \lambda) = \sum_{j=2}^k c_j (1/\lambda)^j \cdot \frac{(\lambda t) - (\lambda t)^j}{1 - \lambda t} = \frac{\lambda t}{1 - \lambda t} \sum_{j=2}^k c_j (1/\lambda)^j - \frac{1}{1 - \lambda t} \sum_{j=2}^k c_j (1/\lambda)^j (\lambda t)^j.$$

Multiplying by $(1 - \lambda t)$ gives

$$(1 - \lambda t) P^T(t; \lambda) = \lambda t \sum_{j=2}^k c_j (1/\lambda)^j - \sum_{j=2}^k c_j t^j,$$

since $(1/\lambda)^j (\lambda)^j = 1$. The first sum is $\lambda t (P(1/\lambda) - c_1(1/\lambda))$, and the second is $P(t) - c_1 t$.

Therefore,

$$\begin{aligned}
(1 - \lambda t)P^T(t; \lambda) &= \lambda t (P(1/\lambda) - c_1(1/\lambda)) - (P(t) - c_1 t) \\
&= \lambda t P(1/\lambda) - c_1 t - P(t) + c_1 t \\
&= \lambda t P(1/\lambda) - P(t).
\end{aligned}$$

—End of Proof of Sub-Lemma 5.1. □

Which, as in the risk-seeking case leads to:

Sub-Lemma 5.2. *Let $0 < r_1 < r_2 < \dots < r_m < 1$ be the positive roots of P , with $r_m < 1$ simple. Fix $\lambda = 1/r_m > 1$. Then*

1. $P^T(t; \lambda) > 0$ for all $t \in [1, \infty)$;
2. $\text{roots}(P^T) = r_1, \dots, r_{m-1}$, each with the same multiplicity as in P .

—Proof of Sub-Lemma 5.2. Because $P(r_m) = 0$, the identity simplifies to

$$(1 - \lambda t) P^T(t; \lambda) = -P(t).$$

(i) For $t \in [1, \infty)$, $\lambda t > 1$, $1 - \lambda t < 0$, $P(t) > 0$ implies $-P(t) < 0$, divided by negative gives $P^T > 0$. (ii) Factor $P(t) = (t - r_m)Q(t)$, then $P^T(t; \lambda) = -(t - r_m)Q(t)/(1 - \lambda t) = Q(t)/\lambda$ after cancellation, removing r_m and preserving other roots/multiplicities.

—End of Proof of Sub-Lemma 5.2. □

We now follow the same basic argument as in the risk-seeking case. The new lottery has the same characteristics (roots all < 1 , $P^T > 0$ on $[1, \infty)$) but one fewer root. Repeating the argument (setting new $\lambda = 1/\text{new largest root} > 1$, assuming $\Delta \not\rightarrow 0$ from the starting wealth, deriving a further $\delta_1 > \delta_0$ where the next normalized trailing ≤ 0 , etc.) reduces roots each time. After finite iterations (in which we lose one root each time), we either must end with all positive coefficients (which leads to a contradiction as above), or to the removal of all roots.

As in the risk-seeking case, we now consider the case in which the iteration leads to a virtual decision with no roots. It must be that for this decision $P(t) > 0$ for $t \in [1, \infty]$. Given there are no roots, then $P(t) > 0$ for all $t > 0$, such that the t -partition of choice B is $\emptyset$. In this decision, the person consistently weakly prefers L_B over L_A . But, this is not possible by Proposition 3 and so we have a contradiction. Therefore $\Delta_w \rightarrow 0$ as $w \rightarrow \infty$.

—End of Proof of Lemma 5. □

D.3.4 Inconsistent Complex Choices

In the same way as we did in Section D.2.3, we now combine the two lemmas above:

Appendix-Proposition 3. *An EUM cannot weakly prefer a complex risk-averse choice L_B over L_A and a complex risk-seeking choice G_B over G_A for all wealth levels.*

—*Proof of Appendix-Proposition 3.* By the risk-averse section, consistent preference for L_B over L_A implies $\Delta_w \rightarrow 0$ as $w \rightarrow \infty$ by Lemma 5. By the risk-seeking section, consistent preference for G_B over G_A implies $\Delta_w \not\rightarrow 0$ as $w \rightarrow \infty$ by 4. This is a contradiction for any increasing utility ($\Delta_w > 0$).

—*End of Proof of Appendix-Proposition 3.* □

D.3.5 Expanding Incompatible Choices Using Tilted Utilities

In the same way as we did in Section D.2.4, we can now use tilted choices and utility functions to expand Proposition 3. To do so, we order t-intervals $A \prec B$ if the supremum of A is strictly less than the infimum of B ; i.e. $\sup A < \inf B$:

Appendix-Proposition 4. *An EUM cannot choose $L_B \succeq L_A$ and $G_B \succeq G_A$ for all wealth levels if $t(L_B \succeq L_A) \succ t(G_B \succeq G_A)$.*

—*Proof of Appendix-Proposition 4.* Suppose that an utility function function exists that rationalizes both decisions. Then, there is a z -tilted utility function that rationalizes both z -tilted utility functions. The tilted first choice will have t -intervals fully below 1, and therefore by Lemma 5, the tilted utility function $\Delta_w^{(\lambda=z)} \rightarrow 0$. The tilted second choice will have t -intervals fully above 1 and therefore by Lemma 3, the tilted utility function $\Delta_w^{(\lambda=z)} \not\rightarrow 0$. Thus, this tilted utility function cannot exist, leading to a contradiction. Thus, the original utility function cannot exist.

—*End of Proof of Appendix-Proposition 4.* □

D.3.6 Final Expansion

There is one more set of choices with non-overlapping t-intervals that we have not addresses: those in which the sets are non-intersection but still “interweaving” For example, consider the case in which one choice has t-intervals of $\{(0, .5], [1.5, \infty)\}$ and the other $[.75, 1.25]$. These sets are non-overlapping, but it is not possible to find a z that separates them.

In this case, we can use the machinery above to iteratively construct tilted utilities and decisions, as well as “leading” decisions, until we end at a situation that leads to a contradiction. We state then the final result:

Appendix-Proposition 5. *An EUM cannot choose $L_B \succeq L_A$ and $G_B \succeq G_A$ for all wealth levels if $t(L_B \succeq L_A) \cap t(G_B \succeq G_A) = \emptyset$.*

—*Proof of Appendix-Proposition 5.* We start with some of the cases removed by the Propositions above. Proposition 3 removes the case in which the t -interval of either decision is $\emptyset$. Proposition 4 removes the case in which some z can split the t -intervals of the decisions. Therefore, we are left with the interweaving case. Note then that the t -intervals of one of the decisions must have at least two connected components.

We now start a process that we will iterate over later. First, as the intervals are non-overlapping and the number of components is finite, one choice must have the right-most component of the union of the intervals (that is, given intervals $[1, 2], [3, 4], [5, \infty)$, $[5, \infty)$ is the right-most component). We label that decision, e.g. $L_B \succeq L_A$, as D_H and the other D_L . Note then that there must be some z such that the right-most connected subset of the $t(D_H)$ lies above z and the entire $t(D_L)$ lies entirely below z . Define $\gamma = 1/z$ and construct the z -tilted utility functions and choices. Then $t(D_L^{(\gamma)})$ lies entirely below 1, while right-most connected subset of $t(D_H^{(\gamma)})$ lies above 1. By Lemma 1, if there is a utility function that rationalizes the D choices for all wealth levels, there must be a utility function $u^{(\gamma)}$ that rationalizes the tilted choices for all wealth levels.

Second, note that $D_L^{(\gamma)}$ is a generalized risk-averse choice. Therefore, $\Delta^{(\gamma)} \rightarrow 0$ by Lemma 5. Third, as $\Delta^{(\gamma)} \rightarrow 0$, the argument in Lemma 4 implies that a person who chooses B in $D_H^{(\gamma)}$ must satisfy

$$\sum_{i=1}^{k-1} \lambda^i L_i(\lambda; D_H^{(\gamma)}) \Delta_{w+i} \leq 0$$

for any $\lambda < 1$, where $L_i(\lambda; D_H^{(\gamma)})$ is constructed using the c_i s in $D_H^{(\gamma)}$.

Let r_m be the highest root in $P(t; D_H^{(\gamma)})$ and set $\lambda = 1/r_m$. r_m must be greater than 1 as the entire right-most component of $D_H^{(\gamma)}$ is greater than 1, so $\lambda < 1$. Sub-Lemma 4.2 then implies that the $t(L_i(\lambda; D_H^{(\gamma)})) = t(D_H^{(\gamma)}) \setminus r_m$. As in Lemma 5, we continue this process until all roots in $D_H^{(\gamma)}$ above 1 are removed.

That is, given that $\Delta_w^{(\gamma)}$ must go to 0, it must be that a person with $u^{(\gamma)}$ will choose B for all wealth levels on the virtual decision $D_{H,new}^{(\lambda)}$ associated with $L_i(\lambda; D_H^{(\gamma)})$, where $D_{H,new}^{(\lambda)}$ has the same t -intervals of $D_H^{(\lambda)}$ except the right-most-interval is removed.

After this process, we conclude that there must exist an increasing utility function $u^{(\lambda)}$ that rationalizes choices $D_L^{(\gamma)}$ and $D_{H,new}^{(\lambda)}$, where the t -intervals of these decisions do not overlap.

Given this, we can continue to iterate on this process of tilting, concluding that the tilted utility must tend to 0, and then using that to construct a new decision which removes the right-most component of the intervals. As some point, we must hit a point in which

the t -intervals of the two decisions can be fully separated by some z . But, then we apply Proposition 4 and conclude that a utility function that rationalizes these decisions cannot exist. Therefore, we have a contradiction and we conclude that no increasing utility function u can exist.

—End of Proof of Appendix-Proposition 5. □

D.3.7 Main Result

Finally, we state our main theorem:

Theorem 1 (Main Result). *An EUM can choose $L_B \succeq L_A$ and $G_B \succeq G_A$ for all wealth levels $\iff \alpha(L_B \succeq L_A) \cap \alpha(G_B \succeq G_A) \neq \emptyset$*

—Proof of Theorem 1. We use α -intervals in the statement to match the paper. However, note that, as discussed above, non-overlapping α -intervals occur iff there are non-overlapping t -intervals. Proposition 5 proves $\Rightarrow$. For $\Leftarrow$, if the alpha-intervals intersect, then by definition there exists an exponential utility function that rationalizes both decisions.

—End of Proof of Theorem 1. □

D.4 Multiplicative Consistency and CRRA

We now consider a multiplicative analogue of the additive consistency studied earlier. This setup is particularly relevant for preferences over lotteries where outcomes scale proportionally with a multiplier, such as in contexts involving relative wealth changes or returns. To ensure utilities are well-defined (e.g., avoiding issues with logarithms at zero), we assume all lottery outcomes are strictly positive real numbers, and final wealth is determined by the scaled outcomes.

D.4.1 CRRA Utility and η -Intervals

We use the constant relative risk aversion (CRRA) utility function as a reference to parameterize curvature in the multiplicative setting. The CRRA utility is defined over positive final wealth $y > 0$ as:

$$v(y; \eta) = \begin{cases} \frac{y^{1-\eta}-1}{1-\eta} & \text{if } \eta \neq 1, \\ \ln y & \text{if } \eta = 1. \end{cases}$$

This form allows for both risk-averse behavior ($\eta > 0$, where v is concave) and risk-seeking behavior ($\eta < 0$, where v is convex). For $\eta = 0$, it reduces to risk-neutrality.

Given two lotteries L_A and L_B with probabilities over positive outcomes $s_0, s_1, \dots, s_k$ where $s_i > 0$ for all i , the scaled lottery $x \cdot L$ for $x > 0$ has outcomes $x \cdot s_i$. The expected utility is $E[v, x \cdot L] = \sum p_i v(x \cdot s_i; \eta)$. We say a person *consistently* prefers L_A over L_B over all multipliers if they prefer $x \cdot L_A$ to $x \cdot L_B$ for every $x > 0$. The η -intervals of a weak preference $L_A \succeq L_B$ is:

$$\eta(L_A \succeq L_B) = \{\eta : E[v(x \cdot s; \eta), L_A] \geq E[v(x \cdot s; \eta), L_B] \text{ for all } x > 0\}.$$

(Note that this is defined for consistency across all scalings, mirroring the wealth-interval consistency in the additive case.)

D.4.2 Mapping from CRRA to CARA via Log Transform

The connection between CRRA (multiplicative) and CARA (additive) arises via a logarithmic transformation. Let $z = \ln y$, so $y = e^z$. Define transformed outcomes $t_i = \ln s_i$ (since $s_i > 0$, $t_i \in \mathbb{R}$). Then, for scaled outcomes $x \cdot s_i = e^{\ln x + t_i}$, the utility in z -space is $u(z; \eta) = v(e^z; \eta)$. Substituting (i) For $\eta \neq 1$: $u(z; \eta) = \frac{e^{(1-\eta)z} - 1}{1-\eta}$ and (2) For $\eta = 1$: $u(z; 1) = z$. This is affine-equivalent to the exponential (CARA) form $r(z; \alpha) = -e^{-\alpha z} / \alpha$ with $\alpha = \eta - 1$. Note that (i) If $\eta > 1$, $\alpha > 0$ (concave CARA), (ii) If $\eta < 1$, $\alpha < 0$ (convex CARA), and (iii) If $\eta = 1$, $\alpha = 0$ (linear).

The expected utility condition $E[v(x \cdot s; \eta), L] = E[u(\ln x + t; \eta)]$ transforms multiplicative scaling ($x > 0$) into additive shifts ($\delta = \ln x \in \mathbb{R}$). Thus, η -intervals map to α -intervals of the transformed lotteries via $\alpha = \eta - 1$.

Corollary 4. *An EUM can choose $x \cdot L_B \succeq x \cdot L_A$ and $x \cdot G_B \succeq x \cdot G_A$ for all wealth levels $\iff \eta(L_B \succeq L_A) \cap \eta(G_B \succeq G_A) \neq \emptyset$*

—*Proof of Corollary 4.* Apply the log transform: Define transformed lotteries with gains $t_i = \ln s_i$ and $q_i = \ln r_i$ (for G lotteries with outcomes $r_i > 0$). The consistency conditions become: Prefer $\delta + T_A$ over $\delta + T_B$ and $\delta + Q_B$ over $\delta + Q_A$ for all $\delta \in \mathbb{R}$, where T, Q are the transformed lotteries. By the mapping, an increasing v exists rationalizing the multiplicative case if and only if an increasing $u(z) = v(e^z)$ exists rationalizing the additive case on the transformed lotteries. From Theorem 5, this holds if and only if the α -intervals intersect for the transformed choices. Since $\eta = \alpha + 1$ maps the intervals directly (preserving intersections), the condition reduces to the η -intervals intersecting for the original (untransformed) choices, as defined.

—*End of Proof of Corollary 4.* □

E Experiment Instructions

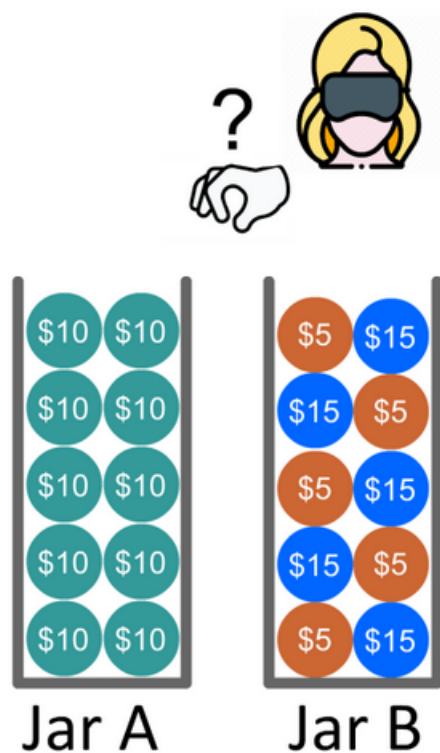
Experiment Instructions (Page 1)

In this study you will be asked to make monetary decisions. These are **real decisions and will determine bonus payments**.

The goal of the study is to determine how you make monetary choices when uncertainty is involved. In every decision:

- You will choose between two Jars: "Jar A" and "Jar B."
- Each Jar will contain balls that give you money.

For example, you could be shown the following two Jars.



Jar A has 10 balls that are all worth \$10. *If you drew a ball out of Jar A at random you would always get \$10.*

Jar B also has 10 total balls, but 5 balls are worth \$5 and the other 5 balls are worth \$15. *If you drew a ball out of Jar B at random you would always get at least \$5, but you would also have a 50% chance of getting \$15.*

In this study, you will have to choose between jars like the ones above. In particular, you will have to choose which jar you would rather RANDOMLY pick one ball from.

However, the jars will be shown to you as tables rather than pictures. For example, instead of the picture above you would be shown:

Jar A	
<u>100% of Balls</u>	
<u>\$10</u>	

Jar B	
<u>50% of Balls</u>	<u>50% of Balls</u>
<u>\$5</u>	<u>\$15</u>

The table and picture above say the **exact same thing**. So please pay careful attention to the percentages and amounts of money involved when making your decisions.

Experiment Instructions (Page 2)

Then you will have to decide how much *more* you value your preferred jar. That is, you will decide what amount of money would need to be taken away from your preferred jar for you to like it exactly as much as the jar you liked less.

For example, suppose you said that you would rather draw a ball from Jar A (shown below).

Jar A	
<u>50% of Balls</u>	<u>50% of Balls</u>
<u>\$15</u>	<u>\$25</u>

Jar B	
<u>50% of Balls</u>	<u>50% of Balls</u>
<u>\$10</u>	<u>\$30</u>

Then you would decide how much **money would have to be taken away from the balls in Jar A** for you to like Jar A exactly as much as you like Jar B.

If you said \$2.50 would have to be **SUBTRACTED** from Jar A for you to like it exactly as much as Jar B, you would be saying that you are **equally happy** to draw a ball at random from either of the following Jars:

Jar A - \$2.50	
<u>50% of Balls</u>	<u>50% of Balls</u>
<u>\$12.50</u>	<u>\$22.50</u>

Jar B	
<u>50% of Balls</u>	<u>50% of Balls</u>
<u>\$10</u>	<u>\$30</u>

Please note, that some answers do NOT make sense. For example, if you said that \$5.00 would have to be subtracted from Jar A for you to like it exactly as much as Jar B, you would be saying you are **equally happy** to draw a ball from either of the following Jars:

Jar A - \$5.00	
<u>50% of Balls</u>	<u>50% of Balls</u>
<u>\$10</u>	<u>\$20</u>

Jar B	
<u>50% of Balls</u>	<u>50% of Balls</u>
<u>\$10</u>	<u>\$30</u>

But Jar B gives you \$10 whenever "Jar A - \$5.00" gives you \$10, and Jar B gives you \$30 when "Jar A - \$5.00" gives you \$20. So you should like Jar B better than "Jar A - \$5.00".

Experiment Instructions (Page 3)

We will ask you several questions like the examples on the previous pages. **To make sure the decision is meaningful, there is a chance each choice will determine your bonus.**

You will be paid according to a special set of rules (detailed below) that ensure it is **always in your best interest to answer accurately and honestly.**

It is also in your best interest to tell us precise amounts: that is, if you would be equally happy with \$4.40, please say \$4.40 rather than \$4.

Details (Not tested in comprehension questions): For every 2-part decision you make, there will be a 50% chance that you will be given a random draw from your preferred Jar with no money subtracted (a computer will randomly determine the outcome of a draw). There will also be a 50% chance that the computer will randomly choose a "staying price" for your decision.

- If the staying price is more than the amount you said would need to be subtracted from your preferred Jar for you to be equally happy to draw from your non-preferred Jar, your potential bonus will be the outcome of a random, computerized draw from your non-preferred Jar.
- If the staying price is less than the amount you said would need to be subtracted from your preferred Jar to be equally happy to draw from your non-preferred Jar, your potential bonus will be equal to the outcome of a draw from your preferred Jar minus the staying price.